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A Mixed-Integer Nonlinear Programming Model for Integrated Feeder Routing and Conductor Selection in Rural Distribution Systems

Hassan Rahman^{1*}, Tariq Mahmood¹

Abstract

This study addresses the problem of cost-optimal expansion planning for rural radial distribution networks via a mixed-integer nonlinear programming (MINLP) formulation that jointly determines feeder routing and conductor type selection. The developed method was evaluated using standard 9-node and 25-node test networks, with performance benchmarked against minimum spanning tree (MST)-based procedures and two metaheuristic techniques (sine-cosine algorithm and tabu search). The application of the MINLP approach led to significant reductions in overall expenditure. In the 9-node test case, the total cost was reduced from USD 131,819.33 (obtained via MST-TSA) to USD 77,129.34, resulting in savings of USD 54,689.99, equivalent to 41.48%. Energy loss costs alone decreased from USD 111,746.73 to USD 63,764.12, a drop of USD 47,982.61 or 42.94%. In the 25-node network, overall costs fell by more than 65%, from USD 371,516.59 to USD 128,974.72, while energy loss expenses decreased by USD 210,057.16 (61.06%). Although conductor-related capital expenditure increased under the MINLP solution, the strategy generated significant long-run economic benefits by markedly reducing operating expenses. In contrast to conventional strategies that address topology definition and conductor sizing in separate stages, the proposed framework integrates these decisions, thereby ensuring globally optimal configurations. These outcomes highlight the method's ability to scale and its practical value for strategic, long-horizon planning in sophisticated distribution grids. All numerical experiments were conducted in Julia (v1.10.2), leveraging JuMP (v1.21.1) together with the BONMIN solver.

Keywords Distribution system planning, Network design, Mixed-integer nonlinear programming, Cost minimization, Energy losses, Conductor selection

*Correspondence:

Hassan Rahman
hrahman@uok.edu.pk

¹ Department of Energy Systems and Optimization, Faculty of Engineering, University of Karachi, Karachi, Pakistan

Introduction

General Context

The creation of electricity distribution infrastructure in rural areas is indispensable for stimulating regional economic activity, raising living standards, and ensuring the delivery of essential services to remote areas [1, 2]. Providing reliable power to rural areas remains one of the principal challenges confronting electric utilities and national authorities worldwide, largely because service must reach small, widely scattered populations [3, 4]. Rural grids differ markedly from their urban counterparts and therefore demand careful design aimed at restraining capital outlays while preserving

acceptable levels of service quality and operational efficiency—particularly in light of the sharply increased unit costs associated with constructing lines across large, sparsely inhabited expanses [5, 6].

Efforts to extend electrification to isolated populations stem from broader objectives of achieving universal energy access, bolstering agricultural productivity and small-scale industry, and catalyzing social and economic progress in marginalized regions [7]. At the same time, rugged landscapes, ecological sensitivities, and severe budgetary restrictions necessitate the adoption of more advanced, optimization-driven planning tools capable of producing affordable yet technically robust network designs [8].

Motivation

The pronounced spatial dispersion typical of rural settlements makes defining an efficient network layout a highly intricate multi-criteria optimization challenge that involves technical performance, economic feasibility, and environmental considerations [9]. **Figure 1** underscores the transformative potential of supplying electricity to rural districts and its close connection to broader developmental goals and greater social cohesion. Reliable electric supply supports delivery of indispensable services—among them schooling, basic healthcare, and digital connectivity—thereby substantially elevating living conditions and creating conditions conducive to local economic expansion [10].



Figure 1. Design and implementation of an electrical distribution network to provide reliable energy access to rural communities. This illustration was created using DALL-E 3 (OpenAI) from a prompt developed by the authors. The image does not reproduce or adapt any existing copyrighted material.

Effective planning and enhancement of rural distribution systems require powerful mathematical optimization frameworks that systematically identify the most advantageous feeder pathways to serve every end user reliably [11]. Many legacy planning practices continue to depend on manually specified topologies or relatively simple heuristic rules, approaches that frequently fall short of delivering either the lowest possible cost or the most robust technical performance [1]. Formulating comprehensive optimization models that explicitly treat route choices as decision variables enables curtailment of capital spending on infrastructure while still complying with all electrical demand and operational limits.

Modern frameworks based on mixed-integer nonlinear programming (MINLP) enable the simultaneous determination of the best conductor types and the most favorable routing arrangements, leading to superior energy-transfer efficiency and better accommodation of challenging topographical and environmental conditions [12, 13].

Literature review

The problem of optimal distribution system planning, which includes both feeder path selection and conductor sizing, has received considerable attention in the technical literature. This section reviews several of the most pertinent studies in this domain.

In Lavorato *et al.* [14], the researchers investigated the incorporation of radiality constraints into optimization frameworks for distribution networks, underscoring their critical importance in both planning and operational contexts, given the inherently radial structure of these systems. Their work included an extensive literature survey, a detailed critical evaluation, and a structured methodology for embedding these constraints into mathematical models. The study showed that radiality constraints can be formulated and implemented efficiently for applications in network reconfiguration and expansion planning, while preserving the desired topological properties. Moreover, the authors generalized these constraints, broadening their utility across a range of power distribution optimization tasks.

Ponce *et al.* [15] examined distribution system expansion in response to rising electricity consumption in urban and industrial environments. The authors introduced a multi-criteria decision framework based on the CRITIC technique to optimize network layouts by simultaneously considering voltage quality, active power losses, current magnitudes, and conductor investment expenses. The procedure was applied to the IEEE 34-bus test case using Matpower within the Matlab environment, producing a decision matrix encompassing 210 candidate solutions. Findings indicated that the method successfully achieves a balanced trade-off among service quality, operational efficiency, and economic performance in distribution planning.

Another relevant contribution [1] focused on the cost-effective expansion of medium-voltage AC distribution networks tailored to rural settings, employing a heuristic strategy. Feeder routing was determined using a minimum spanning tree (MST) formulation, with initial conductor sizes assigned based on the highest anticipated load currents. A tabu search algorithm (TSA) subsequently improved the solution by performing three-phase power flow calculations across different load scenarios. Experiments conducted on 9-node and 25-node test feeders confirmed the approach's ability to lower overall planning expenditures, with the heuristic phase successfully identifying at least 70% of the optimal conductor sizes ultimately selected, thereby accelerating and enhancing the TSA refinement step.

In Lavorato *et al.* [8], the authors propose a constructive heuristic procedure to address the challenging mixed-binary nonlinear programming formulation inherent in distribution system expansion planning. The algorithm featured a local search improvement stage, combined with a specialized branching mechanism, to enhance solution accuracy. A sensitivity-based metric, obtained from a relaxed version of the planning model, directed the incremental addition of lines or substations. Validation on two standard benchmark networks plus one actual utility system confirmed the reliability and effectiveness of the proposed technique.

Li *et al.* [16] introduced a bi-objective model for distribution network expansion planning that jointly considers system reconfiguration and reliability performance. The solution procedure combined the non-dominated sorting genetic algorithm II with heuristic enhancements to tackle the embedded subproblem efficiently. When evaluated on the IEEE 33-bus and 70-bus test systems, the method achieved computation times that were more than 200 times faster than those of prior techniques.

In Silveira *et al.* [17], the researchers studied the distribution network reconfiguration task, which involves adjusting tie switches to maximize technical and economic benefits, particularly minimizing power losses. They conducted a rigorous comparison of conventional optimization formulations and metaheuristic strategies, using standardized performance

indicators to control for biases arising from implementation details or computing platforms. Tests on benchmark networks of 33, 136, and 417 buses showed that linear and conic programming approaches handled small- and medium-sized instances efficiently. In contrast, metaheuristics proved superior for large-scale networks owing to significantly reduced computational demands.

The work in Samui *et al.* [18] explored how reliability criteria influence the topology design and associated planning expenditures in radial distribution systems. A direct search method was applied to minimize total investment costs while exploiting Bellman's principle of optimality to limit the number of radial configurations evaluated, thereby improving computational performance. Reliability metrics were computed to compare alternative feeder arrangements, and the procedure was tested across scenarios involving different substation counts. The analysis provided a valuable understanding of the trade-off between economic optimality and service reliability in multi-substation radial networks, validating the practicality of the proposed methodology.

Finally, Ganguly *et al.* [19] proposed an innovative dynamic programming technique for multi-objective distribution system planning. The method simultaneously optimizes feeder routing and conductor sizing by minimizing two primary objectives: (i) the sum of capital and operational expenses (including new feeder/substation installation, maintenance, and energy losses) and (ii) reliability-related costs (encompassing undelivered energy, repair expenditures, and customer interruption damages). A dynamic programming algorithm was constructed to produce a set of Pareto-efficient solutions through weighted aggregation of the objectives. The technique was assessed on 21-node, 54-node, and 100-node test systems, with a comparative evaluation against multi-objective evolutionary algorithms confirming its superior performance.

Joshi *et al.* [20] introduced a straightforward computational procedure tailored for distribution system planning across extensive geographical regions, with primary emphasis on determining ideal substation sites and feeder pathways. The core goal was to identify the most advantageous substation positions by minimizing overall expenditures while strictly enforcing a radial network topology. Additionally, the framework incorporated feeder routing decisions involving four distinct conductor types, thereby improving economic performance. Departing from conventional practices that restrict designs to a single conductor type throughout the network, this technique allows the assignment of varied conductors across different segments, yielding additional cost savings. Ultimately, the method achieves an effective compromise between capital investment requirements and long-term operational performance during network expansion initiatives.

Dimovski *et al.* [21] put forward an integrated framework for rural electrification planning that draws on publicly available data sources, geospatial referencing techniques, and a mixed-integer linear programming formulation aimed at minimizing the net present value of the entire electrification project—including capital outlays, operational expenses, maintenance costs, and end-of-life residual values. The procedure is organized into four sequential phases: load forecasting, design of hybrid microgrid configurations, layout of internal distribution networks, and holistic regional-scale optimization. Practical application in the Butha-Buthe district of Lesotho enabled the electrification of 72 dispersed communities, with fully renewable solutions implemented in over 50% of cases. Nevertheless, adopting linear approximations constrains the model's ability to capture nonlinear physical behaviors accurately, and the phased, sequential nature of the network design process may prevent convergence to globally optimal configurations.

Gadelha Filho *et al.* [22] outlined a GIS-supported methodology for rural electrification planning that integrates graph-theoretic principles and terrain analysis to produce feasible, realistic electrical grid layouts. The optimization criterion sought to minimize aggregate capital expenditure for the network, accounting for both route selection and conductor sizing, subject to thermal and voltage constraints. Implementation occurred within the specialized GISEle software platform, which incorporates population clustering for density mapping, Steiner tree construction for preliminary routing, and equilibrated power flow computations. An application to a real-world case in Cavalcante, Brazil, achieved 47% in deployment cost savings compared with purely MST-driven solutions. However, the sequential organization of the optimization workflow—first establishing the topology and subsequently allocating electrical components—can constrain

the exploration of globally superior solutions, especially when routing choices and conductor decisions exhibit strong mutual dependencies.

In contrast, Mahdavi *et al.* [23] developed a mixed-integer second-order conic programming formulation specifically for feeder routing in three-phase balanced distribution networks, which was efficiently solved via the AMLP interface within the CPLEX optimization suite. The principal aim was to reduce daily active energy losses across the system. Numerical experiments conducted on test networks containing up to 203 nodes confirmed the model's ability to achieve meaningful reductions in loss. Furthermore, benchmark comparisons against alternative methodologies underscored the proposed technique's superior solution accuracy and favorable computational performance.

For their part, Helmi *et al.* [24] introduced an optimization strategy based on the Harris hawks optimizer for feeder routing in distribution systems. The focus was placed on minimizing power losses under designated loading scenarios while preserving radial operation. Performance was assessed on balanced test networks comprising 33, 85, and 295 nodes, where the approach exhibited competitive results relative to other metaheuristic procedures. Computational runtime was also evaluated, although no formal statistical testing was conducted to verify the consistency and reproducibility of the obtained solutions.

In Jakus *et al.* [25], the distribution system routing challenge was tackled through a genetic algorithm framework, augmented by a dedicated heuristic repair mechanism that enforces radial topology feasibility for all candidate solutions. The optimization targeted simultaneous minimization of energy losses and the line loading index. The procedure was validated on networks of 33, 1760, and 4400 nodes, demonstrating both effectiveness and scalability. While solution quality was contrasted with prior techniques and execution times were documented, the study omitted any statistical evaluation to confirm the reliability and repeatability of the results.

In this literature review, the following key aspects were observed:

1. Numerous investigations highlight the critical necessity of enforcing radial network configurations within optimization models for distribution planning, as this property ensures both manageable computational complexity and practical operational viability.
2. Approaches including multi-criteria decision frameworks, dynamic programming techniques, and various metaheuristic algorithms have demonstrated strong performance in reconciling economic objectives, service reliability, and operational effectiveness across expansion and reconfiguration tasks.
3. Heuristic procedures, spanning-tree-based methods, and sensitivity-guided strategies remain widely adopted for tackling feeder routing and conductor allocation problems, consistently delivering meaningful cost savings alongside improved network behavior in diverse application contexts.

Novelty and contributions

Although the techniques reviewed in the preceding subsection have significantly advanced distribution network planning, important methodological shortcomings persist. A primary limitation is that many existing methods address network topology design and conductor sizing as separate, independent tasks. This separation often leads to suboptimal network configurations because the strong interdependence between routing decisions and electrical characteristics is not adequately accounted for. Furthermore, even when certain studies include operational aspects such as reliability or load variability, they commonly resort to linear or piecewise-linear approximations. Such simplifications fail to faithfully represent the inherently nonlinear behavior of power flow equations, particularly in medium- and low-voltage distribution systems.

Another significant drawback lies in the widespread dependence on heuristic and metaheuristic algorithms. While these methods can handle large-scale instances effectively, they offer no assurance of global optimality and remain highly

sensitive to parameter settings and starting points. In real-world applications involving unbalanced, three-phase rural networks with multiple conductor choices, these approaches risk missing superior solutions within vast combinatorial search spaces.

Consequently, there is a clear need for a unified, more robust planning framework capable of jointly addressing network expansion, conductor selection, and operational constraints while delivering provably optimal results and maintaining scalability in rural distribution contexts.

In response to these limitations, the present work offers the following principal contributions:

1. A new and holistic mixed-integer nonlinear programming (MINLP) model is introduced that simultaneously optimizes feeder routing and conductor sizing in rural radial distribution networks. In contrast to prior techniques that separate these decisions or employ simplifying approximations, the proposed formulation precisely incorporates the nonlinear power flow relationships. It enforces radial topology constraints, thereby establishing a mathematically sound and fully integrated optimization platform.
2. Numerical evaluations on standard 9- and 25-node test systems—featuring 14 and 42 candidate branches, respectively, along with seven available conductor types—demonstrate that the MINLP approach markedly outperforms leading heuristic methods, notably the tabu search algorithm paired with minimum spanning tree (MST) topology generation [1]. Over a 20-year planning horizon, the model yields significant decreases in both total planning costs and cumulative energy losses.
3. Beyond securing globally optimal solutions, the developed methodology facilitates a thorough examination of the economic trade-offs between upfront capital expenditures and ongoing operational expenses. This capability equips network planners with valuable long-term insights, especially for infrastructure projects in rural and resource-constrained regions. Moreover, reliance on exact optimization techniques ensures transparency, reproducibility, and reliability—attributes highly valued in both research and industry settings.

To efficiently solve the resulting MINLP problem, this study adopts an exact optimization strategy that combines branch-and-bound enumeration with an interior-point method. The implementation leverages modern optimization libraries in the Julia ecosystem via the JuMP modeling interface, delivering strong computational performance and stability even for sizable distribution network instances.

Three important modeling assumptions underpin the scope of this investigation. First, the rural distribution planning problem is defined as a single-supply substation serving all consumers connected to the network. Second, all optimization runs are performed under peak-load conditions—the most demanding operational scenario—ensuring that every selected conductor can safely carry the maximum expected currents while preserving adequate voltage levels and operational integrity. Third, the utility company determines the pool of feasible feeder routes through a detailed geographic and technical feasibility assessment that accounts for terrain features and other site-specific restrictions in the target rural areas.

Materials and Methods

This section outlines the approach developed for the optimal expansion planning of rural distribution networks. It begins by presenting a complete mixed-integer nonlinear programming (MINLP) model that simultaneously determines feeder routes and conductor types, accounting for both discrete routing decisions and the system's nonlinear electrical behavior. The formulation includes power flow relations, operational and technical limits, and cost-based objectives, all evaluated over an extended planning period. Next, the numerical solution procedure for the resulting MINLP is explained; it employs a branch-and-bound algorithm combined with an interior-point method to handle the nonlinear components effectively.

The entire methodology was implemented in Julia, using the JuMP algebraic modeling framework and the BONMIN optimization solver.

Mathematical formulation

The central goal of distribution network expansion planning is to identify the most appropriate feeder paths that ensure efficient power delivery to all loads while simultaneously selecting the best conductor sizes for those paths [26]. In addition, optimal route selection for electrical distribution grids seeks to establish a new radial topology that reduces active power losses and enhances overall system performance [26, 27]. This task falls into the class of MINLP problems because it uses binary variables to determine whether a particular distribution branch should be built. Separately, the conductor sizing problem in distribution systems focuses on minimizing the combined capital expenditure for conductors and the operational costs [28]. Like the routing problem, it is typically formulated as an MINLP, with binary decision variables indicating the selection of a specific conductor type for each branch.

The objective function and the complete set of constraints that define the distribution network expansion optimization model are detailed in the following subsections.

Objective function

The objective function is designed to strike an appropriate balance between economic and technical performance by minimizing the total costs arising from conductor installation and annual energy losses. The specific objective function adopted in this study is given below:

$$\min Z_{\text{cost}} = \alpha (\beta Z_{\text{loss}} + Z_{\text{inv}}), \quad (1)$$

$$\alpha = \left(\frac{t_a}{1 - (1 + t_a)^{-N_t}} \right), \quad (2)$$

$$\beta = \left(\sum_{ieT} \left(\frac{1 + t_e}{1 + t_a} \right)^i \right), \quad (3)$$

Where Z_{cost} denotes the total annualized cost of system operation, obtained by aggregating the yearly energy loss cost (Z_{loss}) and the annualized conductor investment cost (Z_{inv}). The detailed expressions for these two cost components appear in Eqs. 4 and 5. The coefficient α serves to annualize both the energy loss expenditures and the conductor capital costs from the network operator's perspective. At the same time, β incorporates the projected escalation in energy prices over the planning period. The parameter i_a represents the internal rate of return, N_t indicates the number of years in the planning horizon, and i_e reflects the annual growth rate of energy prices. The terms Z_{loss} and Z_{inv} are computed as follows:

$$Z_{\text{loss}} = C_{\text{kWh}} T \text{Re} \left\{ \sum_{l \in L} V_l^* J_l^* \right\}, \quad (4)$$

$$Z_{inv} = \sum_{l \in L} (\sum_{c \in C} C_c Y_{lc}) L_l, \quad (5)$$

Where C_{kwh} stands for the average price per kilowatt-hour of energy; T equals the total number of hours in a year (i.e., 8760); V_l is the voltage drop along distribution line l ; J_l is the current carried by distribution line l ; C_c denotes the installation cost per kilometer for conductor type c ; L_l is the length (in kilometers) of distribution line l ; Y_{lc} is the binary variable that determines whether conductor type c is assigned to line l ; and the index sets L and C represent, respectively, the set of candidate lines available for construction and the set of available conductor types.

Set of constraints

The mathematical restrictions that shape the optimal distribution system expansion must faithfully obey Kirchhoff's circuit laws in addition to respecting the allowable voltage deviation limits and the maximum thermal loading of each branch. The full collection of constraints defining this optimization model appears in the following subsections.

Current balance at the system nodes

Current conservation at every node k of the distribution grid is enforced through the equality constraint shown in [Eq. 6](#):

$$I_k^g - I_k^d = \sum_{l \in L} A_{kl} J_l, \quad \{k \in N\}, \quad (6)$$

Where I_{gk} stands for the current supplied from the substation situated at bus k ; I_{dk} indicates the current withdrawn by the load at bus k ; A_{kl} is the corresponding entry in the node-to-branch incidence matrix connecting node k with branch l ; and N represents the complete set of buses in the network.

[Eq.6](#) constitutes a straightforward implementation of Kirchhoff's current law (KCL) applied node by node. Physically, it states that the algebraic sum of all currents arriving at node k (from the substation plus currents entering via connected branches) minus all currents departing from node k (to supply the load plus currents leaving via branches) must equal zero. This relation can be equivalently viewed as a nodal current-flow equilibrium condition, in which the net branch current exchange perfectly balances local generation and demand.

Voltage drop across the distribution lines

The voltage magnitude difference along branch l , which links sending node k to receiving node j , is governed by the equality constraint in [Eq. 7](#):

$$V_l = X_l \sum_{k \in N} A_{kl} V_k = L_l \sum_{c \in C} Y_{lc} Z_c J_l, \quad \{l \in J\}, \quad (7)$$

Where V_k denotes the voltage at bus k ; X_l is the binary variable signaling the presence ($X_l = 1$) or absence ($X_l = 0$) of branch l ; and Z_c is the series impedance associated with the conductor type c assigned to that branch.

Eq. 7 maintains electrical consistency for the voltage drop across branch l by equating two alternative yet equivalent expressions:

- Voltage difference between terminals: The drop equals the difference between the voltage at the upstream node k and the downstream node j (left-hand side of the equation).
- Ohm's law expression: The drop is also given by the product of the branch's effective impedance and the current traversing it (right-hand side).

In the model formulation, the binary variable X_l indicates whether the branch is active. If $X_l = 1$, the branch is built, the voltage-drop relation is enforced, and the drop V_l can be interpreted either as the nodal voltage difference or as the impedance-current product using the chosen conductor's impedance Z_c . If $X_l = 0$, no branch exists, forcing V_l to zero and eliminating any possibility of current flow, which correctly simulates physical disconnection. The binary variable Y_{lc} , in turn, designates which conductor type c is installed on branch l (only one type per branch when constructed), thereby determining the impedance Z_c while simultaneously satisfying both voltage regulation and thermal loading constraints.

Load current at the demand nodes

The current absorbed by loads at demand buses is computed via the equality constraint given in **Eq. 8**. Assuming constant-power load behavior, Tellegen's second theorem permits expressing the relationship among the complex power demand, nodal voltage, and load current at node k in the following form:

$$S_k^{d,*} = V_k^* I_k^d, \quad \{k \in N\}, \quad (8)$$

Where S_k^d is the prescribed complex power demand at node k (a fixed, known input value), and the superscript $\star$ indicates the complex conjugate of the quantity inside the parentheses.

Voltage regulation

The constraint formulated in **Eq. 9** governs the allowable voltage variation throughout the distribution network. The voltage magnitude at each bus k is required to stay within the acceptable range defined by the utility's regulatory standards:

$$V^{\min} \leq |V_k| \leq V^{\max}, \quad \{k \in N\}, \quad (9)$$

Where $V^{\min}$ and $V^{\max}$ denote the lower and upper bounds on voltage magnitude, respectively, as prescribed by the network operator's operational guidelines.

Thermal limit of the conductors

The inequality constraint expressed in **Eq. 10** ensures that the current magnitude in branch l does not surpass the thermal capacity of the conductor type c chosen via the binary variable Y_{lc} :

$$|I_l| \leq \sum_{c \in C} I_c^{\max} Y_{lc}, \quad \{l \in L\}, \quad (10)$$

Where $I_c^{\max}$ represents the highest permissible current for conductor type C . It is worth noting that when a branch l is not selected (i.e., $X_l = 0$), the current through that branch becomes zero, automatically satisfying the thermal limit constraint in **Eq. 10**.

Substation node conditions

The equality constraint in Eq. 11 specifies that current injection into the system occurs exclusively at the substation bus (the slack bus), while all other nodes serve solely as load points. Complementing this, the equality constraint in Eq. 12 fixes the voltage at the substation bus to the nominal system value, conventionally taken as 1 per unit (p.u.):

$$I_k^g = 0, \quad k \neq \text{slack node} \quad (11)$$

$$V_k = V_{nom}, \quad k = \text{slack node}. \quad (12)$$

Radial system topology

The constraint in Eq. 13 ensures full connectivity across the distribution network by requiring each node to have at least one incident branch, thereby eliminating the possibility of isolated buses or disconnected portions of the grid. Concurrently, the equality constraint in Eq. 14 enforces that the total count of selected branches equals $n_n - 1$, which mathematically ensures the formation of a tree-like (radial) structure without loops:

$$\sum_{l \in L} |A_{kl}| X_l \geq 1, \quad \{k \in N\}, \quad (13)$$

$$\sum_{l \in L} X_l = n_n - 1, \quad (14)$$

Where n_n corresponds to the total number of nodes in the network.

It is important to emphasize that the radial topology requirement is incorporated because the overwhelming majority of rural distribution networks are designed and operated in this manner, in full compliance with technical standards and operational policies set forth by utilities [29]. Radial layouts offer significant advantages: they simplify fault protection schemes, reduce overall capital expenditure on infrastructure, and facilitate easier system operation and maintenance [30, 31].

Conductor selection

The constraint formulated in Eq. 15 guarantees that precisely one conductor type c is assigned to each distribution branch l whenever the branch is selected via the binary variable X_l . This rule prevents the illogical assignment of multiple conductor sizes to the same branch and ensures that no conductor is allocated to a branch that remains unbuilt:

$$\sum_{c \in C} Y_{lc} = X_l, \quad \{l \in L\}, \quad (15)$$

It should be noted that the mathematical framework outlined from Eq. 1 through Eq. 15 lends itself readily to extensions that accommodate the integration of distributed generation (DG) sources into distribution grids. To incorporate DG, additional constraints must be introduced to account for their effect on nodal power/current balance. In particular, the current balance relation at each node—presented in Eq. 6—would need to be revised to include current injections from DG units. This requires defining a new term, analogous to the load current expression in Eq. 8, to compute the current

contributed by each DG based on its specified complex power output. Furthermore, it becomes essential to add upper and lower bounds on the active and reactive power produced by these sources, thereby ensuring that the optimization respects the physical and operational limits of the generation equipment.

Beyond DG integration, the model can also be adapted to handle demand-side uncertainty or temporal load fluctuations. One practical strategy is to generate a set of representative demand scenarios derived from probabilistic predictions or historical patterns. These scenarios can then be embedded into the formulation, most naturally by treating demand as a variable in Eq. 8 and introducing time as an explicit index set within the model. Including the temporal dimension enables the identification of network designs that maintain both economic attractiveness and technical viability across a range of load conditions. Such enhancements would considerably broaden the model's usefulness in realistic planning settings, where both supply from distributed sources and consumer demand display pronounced time-varying and location-dependent behavior.

With these modifications in place, the proposed framework would be well-suited to support the planning of contemporary active distribution systems, delivering optimized feeder routing and conductor sizing while explicitly addressing uncertainties in generation and load.

Impact of conductor and route set reduction on computational complexity

The distribution expansion planning model defined by Eqs .1 to 15 constitutes a mixed-integer nonlinear programming (MINLP) problem, arising from the coexistence of binary variables (governing branch construction and conductor assignment) and nonlinear expressions (particularly the load current calculation at demand nodes in Eq. 8). The task of simultaneously selecting a subset of branches that form a radial tree and assigning one conductor type to each selected branch creates an exponentially large set of feasible configurations. Every potential solution corresponds to a combination structured as follows:

$$C(n_l, n_n - 1) \cdot n_c^{n_n - 1} = \frac{(n_l)!}{(n_l - n_n + 1)! (n_n - 1)!} \cdot n_c^{n_n - 1}, \quad (16)$$

Where n_l denotes the total number of candidate branches available for construction, and n_c represents the number of distinct conductor types offered. For illustration, consider a 9-node test system with 14 candidate branches and 7 conductor options per branch, yielding roughly 1.7312×10^{10} possible combinations. In a 25-node benchmark with 42 candidate branches and the same 7 conductor choices, the figure rises to approximately 6.7762×10^{31} combinations. Extending the example to a 100-node network with 180 possible branches produces an astronomical $\approx 1.7158 \times 10^{136}$ combinations. These figures vividly demonstrate the combinatorial explosion that characterizes the problem.

Consequently, the time needed to solve the model grows exponentially as a function of the node count n_n , the number of candidate branches n_l , and the number of conductor types n_c . More precisely, the size of the search space is governed by (i) the number of ways to select exactly $n_n - 1$ branches from n_l candidates while preserving radiality, multiplied by (ii) the assignment of one of n_c conductor types to each of those selected branches. This explosive growth is the primary reason the problem is classified as NP-hard—meaning no polynomial-time algorithm is expected to exist for exact solution in the worst case—and explains why brute-force enumeration or even rigorous exact methods become impractical for larger networks.

The computational challenge is further aggravated by the nonlinear constraints, which include Kirchhoff's laws, voltage magnitude bounds, and conductor thermal limits. Although exact optimization algorithms (such as branch-and-bound combined with nonlinear interior-point solvers) are required to guarantee global optimality, their resource demands escalate rapidly with problem size.

Solution strategy

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graph TD; A[Distribution network expansion problem] --> B[Branch-and-Bound]; B --> C[Divides the problem into subproblems (branch nodes)]; C --> D[Conductor Selection (Y_lc)  
Route Selection (X_l)]; D --> E[Solve NLP subproblems at branch node.  
Objective Function (Z_cost): Equations (1)–(5)  
Constraints Check (I_k^g, J_l, V_l, V_k): Equations (6)–(15)]; E --> F[Subproblems optimal solutions]; F --> D; E --> G[Interior-Point Method]; G --> H[Optimal route and conductor size];
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The flowchart illustrates the proposed Branch-and-Bound algorithm for distribution network expansion. It begins with the "Distribution network expansion problem", which leads into the "Branch-and-Bound" process. This process involves "Dividing the problem into subproblems (branch nodes)". The algorithm then proceeds to "Conductor Selection (Y_{lc})" and "Route Selection (X_l)". These steps lead to "Solve NLP subproblems at branch node", which includes the "Objective Function (Z_{cost}): Equations (1)–(5)" and "Constraints Check (I_k^g, J_l, V_l, V_k): Equations (6)–(15)". The "Interior-Point Method" is used to solve these subproblems. The "Subproblems optimal solutions" are then used to refine the "Conductor Selection" and "Route Selection". The final output is the "Optimal route and conductor size".

As depicted in **Figure 2**, the B&B algorithm excels at managing the discrete decisions involved in route construction and conductor assignment, systematically exploring only those integer-feasible regions while progressively tightening bounds and pruning unpromising branches [34]. At every node of the B&B tree, a relaxed version of the original problem—where integer variables are temporarily treated as continuous—is solved via the IPM. This relaxation exploits the convexity

properties of the continuous subproblem, enabling faster and more reliable computation while still guiding the search toward globally optimal integer solutions [35].

The IPM itself is a well-established technique renowned for its strong performance on large-scale nonlinear optimization tasks [36]. Within the B&B framework, it solves a sequence of barrier problems in which logarithmic barrier terms approximate inequality constraints; the barrier parameter is gradually decreased until the solution converges to the optimum of the relaxed problem. The tight coupling of B&B and IPM allows the complex MINLP to be decomposed into a hierarchy of simpler subproblems, leading to improved convergence behavior and higher solution quality [37]. By implementing this hybrid strategy entirely within the Julia ecosystem, the proposed approach successfully addresses the inherent challenges of MINLP models, delivering optimal feeder routing and conductor sizing decisions that simultaneously satisfy feasibility, reliability, and economic objectives for the distribution network.

Table 1 outlines the principal steps of the combined B&B + IPM procedure applied to solve the MINLP model for optimal feeder routing and conductor selection in rural distribution systems.

Table 1. General structure of the solution methodology for solving a generic MINLP problem by combining the B&B method and the IPM.

Step	Description
1. Problem definition	Formulate the MINLP model for simultaneous routing of distribution feeders and conductor selection in radial MV networks. Define the objective function, constraints, and decision variables as integers.
2. Relax integer variables	Transform the MINLP model into an NLP by considering integer variables (such as conductor selection and network topology) as continuous. This allows IPM to solve the relaxed subproblems.
3. Solve the relaxed problem	Apply the IPM to solve the relaxed problem, identifying an initial feasible solution while effectively addressing nonlinear constraints.
4. Check integer constraints	Ensure that the current solution meets all integer constraints (e.g., conductor choices must be discrete).
5. Store feasible solution	If the solution satisfies integer constraints and improves the objective function, store it as a potential optimal solution.
6. Bounding process	Determine whether the current solution improves on the best-known bound; if it does not, discard the branch (pruning).
7. Branching step	If integer constraints are violated, split the problem into smaller subproblems by selecting a branching variable and applying new constraints to enforce integer feasibility.
8. Check for remaining subproblems	If additional subproblems remain, return to Step 2 and repeat the process.
9. Optimal solution found	When no subproblems are left to explore, the best stored solution is considered the optimal routing and conductor selection plan.

In the context of this study, the Julia programming language paired with the BONMIN solver (an open-source solver for mixed-integer nonlinear programming) creates a powerful, flexible platform for addressing optimization challenges in electrical distribution planning. Julia’s exceptional computational speed—driven by just-in-time compilation—and its native

interoperability with cutting-edge solvers make it particularly well suited for demanding, large-scale problems. BONMIN seamlessly merges branch-and-bound enumeration with interior-point methods (and outer-approximation techniques), enabling it to efficiently handle both the combinatorial aspects of route and conductor selection and the nonlinear power-flow constraints expressed in the complex domain [38]. This integrated environment supports accurate modeling and reliable solution of intricate optimization tasks such as the one investigated here.

The key strengths of the adopted solution approach include the following:

1. Julia's just-in-time compilation delivers runtimes that are markedly faster than those of conventional interpreted high-level languages, resulting in superior efficiency when solving large-scale MINLP instances.
2. BONMIN effectively combines branch-and-bound search, outer approximation, and interior-point methods to deliver dependable, high-quality solutions across a wide range of MINLP formulations.
3. Using the JuMP modeling package in the Julia ecosystem, complex nonlinear constraints and integer decision variables can be expressed naturally and intuitively, greatly facilitating advanced power system optimization applications.
4. Both Julia and BONMIN are fully open source, eliminating licensing costs, encouraging community contributions, and allowing researchers to tailor or extend the algorithms better to match the specific characteristics of individual problem classes.

Test Feeder

To evaluate the proposed mathematical model's performance and verify its ability to solve the optimal expansion planning task in distribution grids, two standard benchmark feeders widely referenced in the literature—the 9-node and 25-node systems introduced earlier—were selected for numerical validation. These feeders have served as test cases in earlier studies of the same problem, making them suitable for benchmarking the new approach against published results and for observing its behavior under different network scales, topologies, and degrees of difficulty. As highlighted earlier in Eq. 16, problem complexity increases dramatically with larger numbers of nodes and candidate branches, due to the exponential growth in the number of feasible configuration combinations.

Nine-node test feeder

The 9-node benchmark operates at a nominal line-to-line voltage of 13.2 kV. It contains nine buses and serves eight load points, with a total connected three-phase load of 7625 kW active power and 3692.95 kVAr reactive power. In the context of optimal network expansion, 14 candidate line segments (potential routes) are available for selection, as illustrated in the single-line diagram of Figure 3.

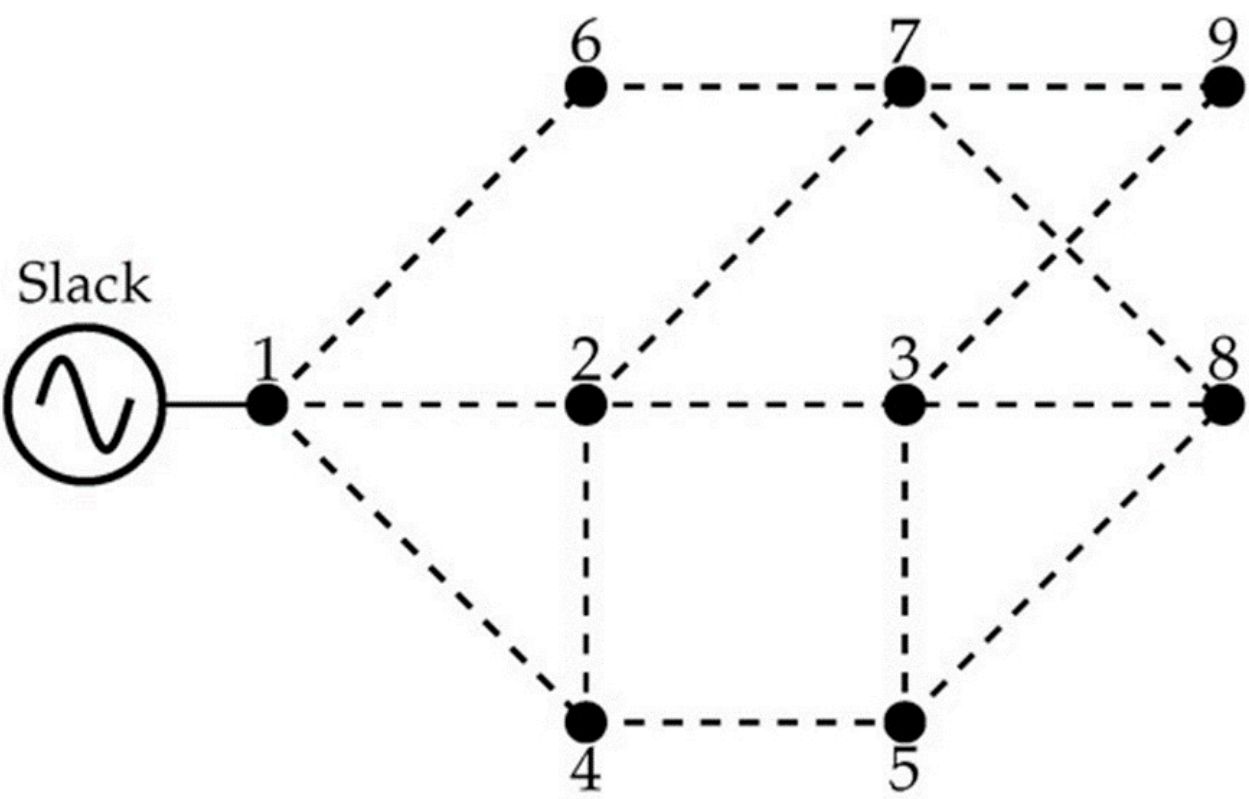


Figure 3. Single-line diagram of the 9-node test system.

Essential input parameters for the 9-node feeder are presented in Table 2 (projected three-phase load demand at each bus) and Table 3 (a list of all candidate branches that may be constructed).

Table 2. Three-phase power consumption at each node of the 9-node test system.

Node <i>i</i>	P _i (kW)	Q _i (kvar)	Node <i>i</i>	P _i (kW)	Q _i (kvar)
2	850	411.6738	6	1500	726.4832
3	750	363.2416	7	500	242.1611
4	925	447.9979	8	850	411.6738
5	1000	484.3221	9	1250	605.4026

Table 3. Possible lines for construction in the 9-node test system.

Line <i>l</i>	Node <i>i</i>	Node <i>j</i>	LI (km)	Line <i>l</i>	Node <i>i</i>	Node <i>j</i>	LI (km)
1	1	2	0.40	8	3	8	0.92
2	1	4	0.85	9	3	9	0.87
3	1	6	0.60	10	4	5	0.65

4	2	3	0.65	11	5	8	1.00
5	2	4	0.65	12	6	7	0.65
6	2	7	0.82	13	7	8	0.80
7	3	5	0.65	14	7	9	0.72

Twenty-five-node test feeder

The 25-node benchmark also uses a 13.2 kV line-to-line voltage level. It includes 25 buses supplying 24 load points, with an aggregate three-phase active power demand of 6375 kW and reactive power of 3087.55 kVAr. For the expansion planning exercise, 42 candidate line segments (possible routes) are considered, as shown in the single-line diagram provided in [Figure 4](#).

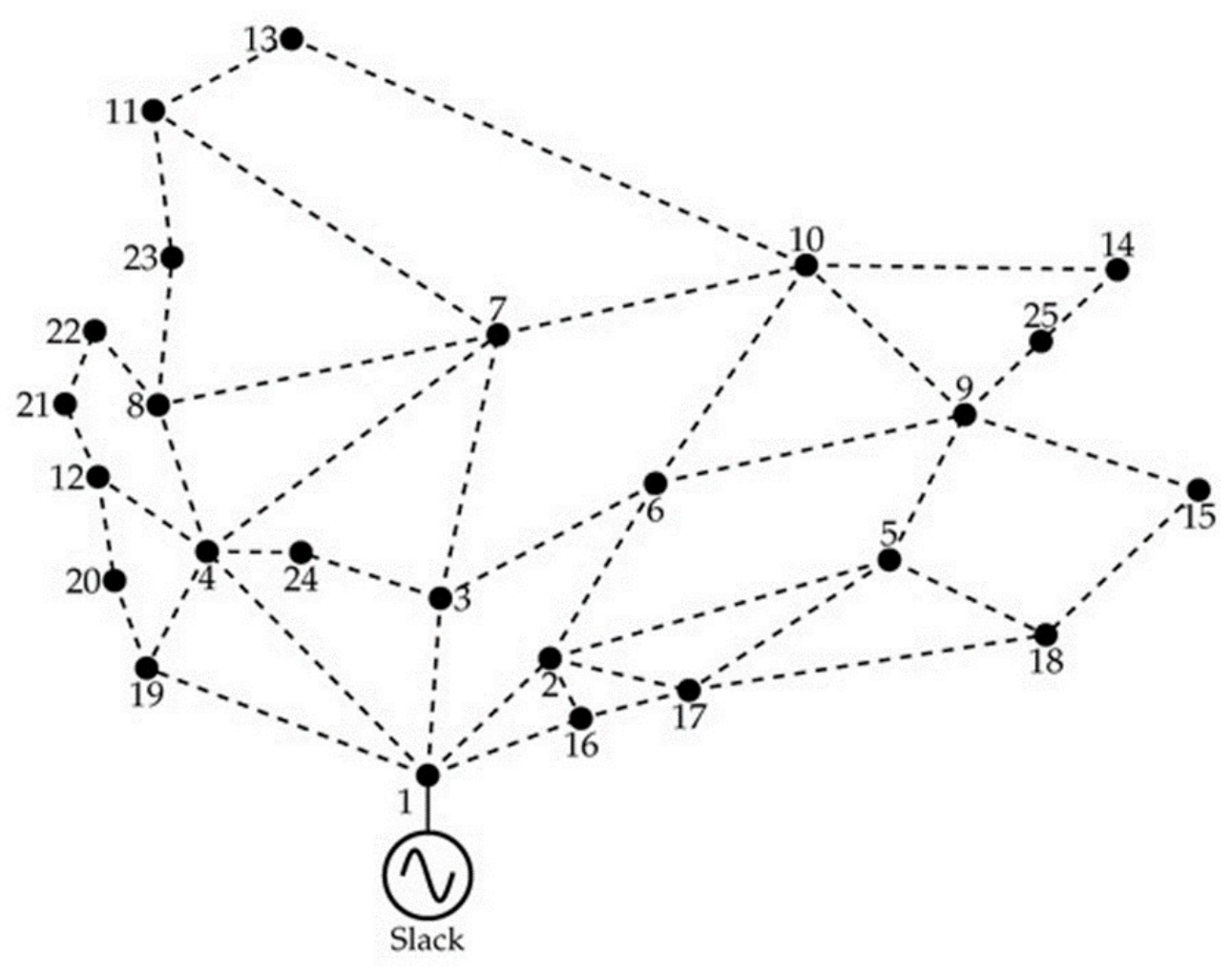


Figure 4. Single-line diagram of the 25-node test system.

The main data characterizing the 25-node feeder are compiled in [Table 4](#) (forecast three-phase load per bus) and [Table 5](#) (complete set of candidate branches eligible for inclusion).

Table 4. Three-phase power consumption at each node of the 25-node test system.

Node <i>i</i>	Pi (kW)	Qi (kvar)	Node <i>i</i>	Pi (kW)	Qi (kvar)
2	625	302.7013	14	250	121.0805
3	400	193.7288	15	250	121.0805
4	250	121.0805	16	375	181.6208
5	250	121.0805	17	200	96.8644
6	125	60.5403	18	100	48.4322
7	250	121.0805	19	250	121.0805
8	250	121.0805	20	100	48.4322
9	625	302.7013	21	150	72.6483
10	400	193.7288	22	100	48.4322
11	250	121.0805	23	200	96.8644
12	400	193.7288	24	250	121.0805
13	250	121.0805	25	75	36.3242

Table 5. Possible lines for construction in the 25-node test system.

Line <i>l</i>	Node <i>i</i>	Node <i>j</i>	LI (km)	Line <i>l</i>	Node <i>i</i>	Node <i>j</i>	LI (km)
1	1	2	2.10	22	1	16	1.50
2	1	3	1.65	23	2	16	1.05
3	1	4	2.20	24	16	17	0.75
4	2	5	2.00	25	2	17	1.05
5	2	6	1.50	26	5	17	1.00
6	3	6	1.75	27	17	18	1.50
7	3	7	1.75	28	5	18	0.75
8	4	7	1.75	29	15	18	1.25
9	4	8	1.00	30	1	19	1.55
10	4	12	1.00	31	4	19	1.00
11	5	9	1.25	32	19	20	0.75
12	6	9	1.50	33	12	20	0.75
13	6	10	1.75	34	12	21	0.50
14	7	10	2.00	35	21	22	0.50

15	7	11	2.00	36	8	23	1.05
16	7	8	1.75	37	11	23	0.50
17	9	15	1.25	38	8	22	0.65
18	9	10	1.75	39	3	24	0.75
19	10	14	1.75	40	9	25	0.45
20	10	13	2.75	41	14	25	0.50
21	11	13	1.75	42	4	24	0.40

Remark 1. Both test feeders depicted in **Figures 3 and 4** were treated as balanced three-phase networks. Accordingly, the analysis relied on their single-phase equivalent models. When normalizing quantities to per-unit (p.u.) values, however, three-phase base quantities were consistently applied for voltage (line-to-line basis) and power (three-phase basis).

Available conductor sizes and additional parametric information

Seven distinct conductor types were offered for assignment in both benchmark systems to allow proper evaluation of the cost objective. The per-kilometer installation costs used in the optimization were drawn from actual procurement and construction data supplied by utility companies and local vendors [1]. **Table 6** lists the technical and economic parameters for each conductor option, including series impedance, maximum allowable current (thermal rating), and unit cost.

Table 6. Conductor sizes available for the optimal distribution network expansion problem.

Caliber (c)	R _c (Ω/km)	X _c (Ω/km)	I _{maxc} (A)	C _c (USD/km)
1	0.80	0.50	75	1500
2	0.72	0.45	100	2000
3	0.65	0.40	125	2750
4	0.60	0.40	150	3000
5	0.55	0.36	175	3500
6	0.45	0.32	200	4200
7	0.40	0.20	225	4700

Likewise, the objective-function values defined in **Eqs. 1 and 5** were evaluated using the system parameters assembled in **Table 7**.

Table 7. Parametric information used for the system cost calculations.

Parameter	Value	Unit	Parameter	Value	Unit
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CkWh	0.1302	USD/kWh	T	8760	hours
ta	10	%	Nt	20	years
te	2	%	Vmin	-10	%
Vmax	+10	%	—	—	—

Results and Discussion

Computational experiments involving the MINLP formulation were executed on a desktop machine with a 12th-generation Intel Core i7-12700T processor (base frequency 1.40 GHz), 32 GB of RAM, and 64-bit Windows 11 Pro. Every script was written in Julia version 1.10.2. The actual MINLP instances were solved using the JuMP modeling package interfaced with the BONMIN solver.

To gauge the quality and reliability of the suggested MINLP approach, its performance was directly compared with the findings reported in Montoya *et al.* [1]. That prior investigation simplified the inherently complex expansion-planning task by splitting it into two independent phases. During phase one, the authors determined feeder pathways using minimum spanning tree (MST) algorithms. This well-known graph-theory method links every node while minimizing the aggregate length of the selected edges, thereby producing a loop-free radial layout at the lowest possible total connection distance [39, 40]. In phase two, conductor diameters were optimized on top of the fixed MST-derived skeleton employing two popular metaheuristics: the sine-cosine algorithm (SCA) and tabu search (TSA).

Benchmarking the new MINLP formulation against established techniques is indispensable for confirming its correctness, assessing solution superiority, and quantifying improvements over recognized methods in the field. Such side-by-side evaluation bolsters trust in the approach's real-world viability and helps delineate precisely where it outperforms—or falls short of—current state-of-the-art procedures.

9-Node test feeder

Application of the proposed MINLP model to the 9-node benchmark yielded the outcomes displayed in Figure 5 and Table 8. Figure 5 juxtaposes the optimal feeder layout computed by the MINLP against the radial configuration generated in Montoya *et al.* [1] through the MST procedure.

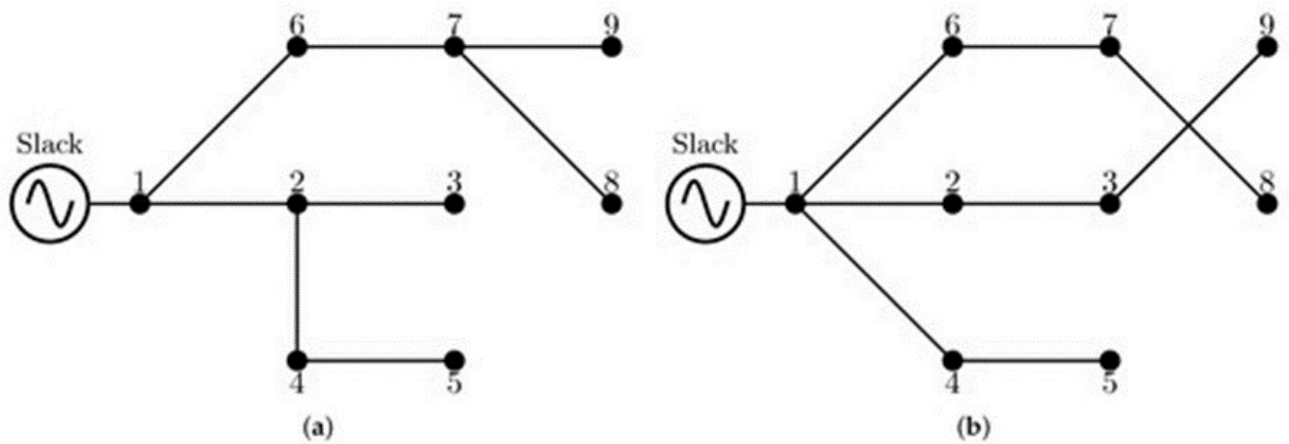


Figure 5. Optimal route selection for the 9-node system: (a) system with the radial topology provided by the MST approach (shortest distance) and (b) system with the radial topology provided by the MINLP model.

Table 8. Numerical results for the 9-node system.

Method	Caliber	Zcost (USD)	Zloss (USD)	Zinv (USD)
MST-SCA	[6,7,1,3,1,4,1,1]	132,699.1306	112,471.8028	12,467.5000
MST-TSA	[7,7,1,2,1,4,1,1]	131,819.3324	111,746.7327	12,180.0000
MINLP	[7,7,7,7,7,6,7,2]	77,129.3388	63,764.1247	23,224.0000

Table 8 presents the quantitative performance metrics for the 9-node case, comparing the MINLP results with those from three competing optimization strategies. Reported figures for each technique include the conductor type assigned to every built branch (**Figure 5**), the overall planning cost (Zcost), the component attributable to energy losses (Zloss), and the expenditure on conductors alone (Zinv).

Inspection of the configurations in **Figure 5**, together with the data in **Table 8**, leads to the following conclusions:

- The MST-derived topology achieved a noticeably shorter cumulative conductor length (5.12 km) than the layout selected by the MINLP model (5.47 km). This length advantage translated into substantially lower upfront conductor expenditures under the MST-based procedures: roughly USD 12,467.50 for MST-SCA and USD 12,180.00 for MST-TSA, compared with approximately USD 23,224.00 required by the MINLP solution—nearly double the amount. Importantly, however, the MST strategies focused solely on reducing total line length and ignored electrical performance indicators, such as active power losses.

Furthermore, conductor choices in the MST-SCA and MST-TSA solutions showed considerable diversity across branches, underscoring an optimization bias toward the cheapest possible investment, achieved primarily through route-length minimization. In sharp contrast, the MINLP model exhibited a clear tendency to install larger-diameter conductors more consistently throughout the network (with types seven and six dominating), reflecting a deliberate strategy that trades elevated initial capital costs for markedly lower future energy-loss expenses.

- The configuration derived from the MINLP model achieved a superior expansion plan by simultaneously accounting for both financial considerations (capital expenditures and operational expenses) and electrical performance requirements (voltage regulation and branch loading limits). As shown in **Figure 5b**, this approach deliberately incorporated additional branch circuits, which facilitated a more balanced current distribution across the network and, as a direct result, led to a substantial decrease in overall energy dissipation. Specifically, the MINLP solution recorded energy loss costs of USD 63,764.12—a reduction of USD 48,707.68 (43.31%) relative to the MST-SCA outcome and USD 47,982.61 (42.94%) relative to the MST-TSA outcome.

- With respect to the aggregate system cost, the MINLP model delivered the lowest figure (USD 77,129.34), corresponding to savings of approximately USD 55,569.79 (41.88%) against MST-SCA and USD 54,689.99 (41.48%) against MST-TSA.

- Concerning execution times, the MINLP model reached proven global optimality in roughly 15.08 seconds—a duration very close to that of MST-TSA (14.35 s) and markedly shorter than the 28.76 seconds required by MST-SCA. These timings illustrate that, for small-to-medium-sized networks, the exact optimization method remains highly competitive in terms of runtime while guaranteeing the mathematically best possible solution.

Taken together, these findings confirm that the MINLP formulation produces a considerably more economical overall design by dramatically cutting energy loss expenditures (Zcost), even though it demands a larger upfront outlay for conductors (Zinv). This underscores the clear benefit of jointly optimizing network layout and conductor selection within a single framework, rather than tackling them sequentially as in the MST-SCA and MST-TSA procedures. From a utility planning standpoint, the elevated initial conductor cost under the MINLP approach is more than offset over the long term by the sustained reduction in operational losses, rendering it a far more financially attractive and technically superior option for expanding rural distribution infrastructure.

Twenty-five-node test feeder

The 25-node benchmark displayed trends broadly consistent with those seen in the 9-node case, though the differences were more pronounced owing to the greater network scale and complexity. **Figure 6** compares the radial structure generated by the MST procedure with the one computed by the MINLP model, while **Table 9** details the conductor assignments for each built branch, along with the associated investment costs, energy loss costs, and total costs across the competing methods.

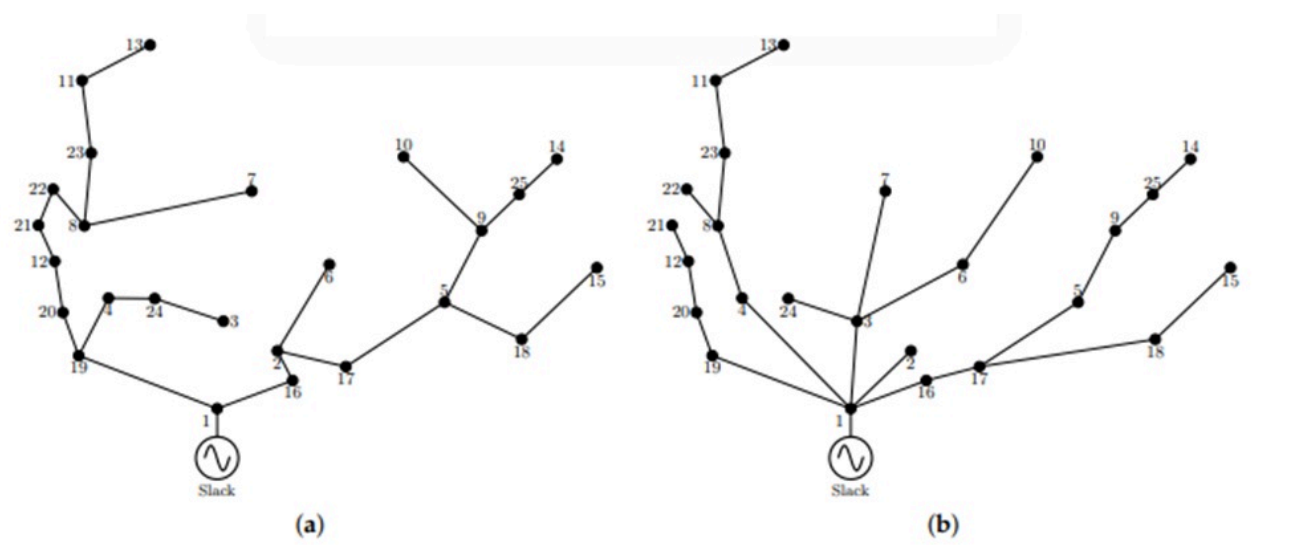


Figure 6. Optimal route selection in the 25-node system: (a) system with the radial topology provided by the MST method (shortest distance) and (b) system with the radial topology provided by the MINLP model.

Table 9. Numerical results for the 25-node system.

Method	Caliber	Zcost (USD)	Zloss (USD)	Zinv (USD)
MST-TSA	[1,1,1,1,1,7,7,6,2,1,1,5,1,2,2,1,1,1,1,1,1,1,1]	371,516.5933	313,238.9341	51,270.0000
MINLP	[1,7,7,1,1,7,6,1,1,7,7,7,1,1,2,1,1,1,1,1,1,1,1]	128,974.7251	103,181.7780	73,045.0000

Examination of **Figure 6** and **Table 9** reveals the following key points:

- Consistent with the smaller test case, the MST-TSA configuration minimized total conductor length to 23.95 km, which kept conductor investment relatively low (USD 51,270) but resulted in exceptionally high energy loss costs (USD 313,238.9341), producing an overall cost of USD 371,516.5933.
- By comparison, the MINLP model allocated greater resources to conductors (USD 73,045), reflecting an extended total line length of 28.65 km. Nevertheless, the topology it selected (**Figure 6**) achieved a dramatic reduction in energy loss

costs (USD 103,181.7780) through more effective power-flow distribution and avoidance of heavily overloaded branches. This yielded a total system cost of approximately USD 128,974.7251.

- Conductor selection in the MST-TSA approach relied heavily on the smallest available sizes (primarily types one and two), with only occasional use of larger ones (e.g., types six, seven, and five) where absolutely required to satisfy basic operational constraints. This pattern indicates a clear focus on curtailing capital spending across most of the network. In sharp contrast, the MINLP model adopted a more uniform strategy of deploying larger conductors (with types seven, six, and two being most prevalent), demonstrating a deliberate emphasis on minimizing cumulative long-term losses, which exert far greater influence on the final objective value, as confirmed by the data in [Tables 8 and 9](#).

In essence, whereas MST-TSA sought to economize on immediate investment by favoring smaller conductors wherever possible, the MINLP model struck a more balanced compromise between capital outlay and loss reduction, ultimately producing a network with enhanced electrical capacity and significantly lower ongoing operational expenses over the entire planning period.

- The resulting reduction in total cost—exceeding USD 188,282.1561, or 51.65% relative to MST-TSA—vividly illustrates the superiority of concurrently optimizing route decisions and conductor sizing through exact mathematical techniques instead of decomposing the problem into isolated stages. By treating both aspects together, the model captures their inherent interdependence, allowing investment choices and loss-minimization goals to be optimized in concert and thereby increasing the probability of identifying the true global optimum. Sequential methods, by contrast—such as fixing the topology first and only then sizing conductors—restrict the search space and frequently converge to locally optimal designs that prioritize short-term savings while underestimating the enduring financial impact of elevated energy losses.

- Additionally, the tabu search algorithm (TSA) exhibited a standard deviation of USD 640.21 across multiple executions on the 25-node system, reflecting noticeable variability in the quality of solutions obtained. The proposed MINLP model, however, produced a standard deviation of zero, confirming its perfect repeatability and deterministic convergence to the identical optimal value in every run.

- In this larger instance, the MINLP solution time rose considerably to 55,546.18 seconds, driven by the exponential increase in the feasible solution space. By contrast, TSA delivered an acceptable (though suboptimal) solution in under 70 seconds. This disparity highlights a classic trade-off between guaranteed optimality and computational feasibility, underscoring the value of future investigations into hybrid algorithms, decomposition techniques, or approximation strategies that could broaden the reach of exact methods to larger, more realistic distribution networks.

The outcomes across both benchmark systems clearly indicate that the MINLP approach delivered major reductions in energy loss expenditures, amounting to approximately USD 47,982.61 (42.93%) for the 9-node feeder and USD 210,057.16 (61.06%) for the 25-node feeder when compared with the MST-TSA benchmark. As a direct consequence, overall planning costs also fell substantially—by roughly USD 54,689.99 (41.48%) in the 9-node case and USD 242,541.87 (65.28%) in the 25-node case—despite the higher upfront expenditure on conductors required by the MINLP solution. From a computational reliability standpoint, the MINLP model consistently produced a standard deviation of zero in both test cases. This complete reproducibility underscores the deterministic character of exact optimization techniques and provides compelling confirmation that the reported solutions represent true global optima. While processing times for the exact method increased noticeably with system size, the resulting designs exhibited markedly lower total costs and superior electrical performance, approaching especially well suited to strategic, long-horizon planning tasks in which achieving the highest possible solution quality outweighs moderate increases in computation time.

These results emphasize the critical importance of jointly optimizing feeder routing and conductor sizing within a unified framework to secure globally efficient network configurations. Moreover, the consistent and strong performance observed on the 9- and 25-node feeders attests to the method's promising scalability, suggesting that—with appropriate computational enhancements—it has potential for application to substantially larger, more operationally complex real-

world distribution grids. Overall, the proposed methodology emerges as a powerful asset for forward-looking infrastructure planning in electrical utilities.

Conclusion

This paper presented a mixed-integer nonlinear programming (MINLP) formulation specifically designed to tackle the optimal expansion planning problem in radial distribution networks, simultaneously determining feeder routes and conductor types. The proposed model was rigorously tested on standard 9-node and 25-node benchmark feeders. In addition, its performance was systematically compared against established techniques that rely on metaheuristic algorithms (SCA and TSA) combined with minimum spanning tree (MST) topology generation.

Numerical experiments demonstrated clear and substantial improvements in both total planning costs and energy loss expenditures when using the MINLP model compared with MST-based metaheuristic strategies. For the 9-node system, total cost declined from approximately USD 131,819.33 (MST-TSA) to USD 77,129.34 (MINLP), yielding savings of USD 54,689.99 or 41.48%. Concurrently, energy loss costs dropped from USD 111,746.73 to USD 63,764.12—a reduction of USD 47,982.61 (42.94%). In the 25-node system, the total cost fell by more than 65%, decreasing from roughly USD 371,516.59 (MST-TSA) to USD 128,974.72 (MINLP). Energy loss costs similarly decreased by USD 210,057.16 (61.06%), moving from USD 313,238.93 to USD 103,181.78. Although the MINLP solution entailed greater initial conductor investment, the considerable long-term savings in energy losses more than offset this additional capital outlay, underscoring the clear economic and technical superiority of integrated optimization over sequential, decoupled approaches.

Looking ahead, several promising directions for extending this work can be identified. First, the model could be generalized to handle unbalanced three-phase distribution networks, explicitly accounting for star and delta load connections as well as the impedance asymmetries arising from non-transposed conductor arrangements. Second, reformulating the problem as a multi-objective optimization task would enable simultaneous consideration of economic, technical (e.g., reliability, voltage quality), and environmental criteria. Third, incorporating distributed energy resources—such as photovoltaic generation, battery energy storage systems, and electric vehicle charging infrastructure—would allow assessment of how increasing decentralization influences optimal expansion decisions. Fourth, integrating heuristic warm-start procedures or hybrid metaheuristic-exact algorithms could substantially enhance computational tractability, potentially enabling application to networks with more than 100 nodes without sacrificing solution accuracy. Finally, addressing demand and renewable generation uncertainty through stochastic programming or robust optimization frameworks would better reflect real operating conditions by incorporating time-varying, scenario-based hourly profiles into the planning process.

Abbreviations

B&B: Branch-and-bound

BONMIN: Basic open-source nonlinear mixed-integer programming

DG: Distributed generation

IMP: Interior-point method

MINLP: Mixed-integer nonlinear programming

MST: Minimum spanning tree

SCA: Sine-cosine algorithm

TSA: Tabu search algorithm

Variables

α : Cost annualization factor

β : Demand growth factor

Akl: Position of the node-to-branch incidence matrix relating node k to line l

Cc: Cost of installing one kilometer of a type C conductor (USD/km)

CkWh: Average energy purchase cost at the substation node (USD/kWh)

ia: Expected internal rate of return (%)

ie: Annual percent growth of the energy cost (%)

Imaxc: Maximum current that can flow through a type C conductor (A)

Idk: Current consumption at bus k (A)

Igk: Current injection by a substation connected at bus k (A)

Jl: Current flowing through distribution line l (A)

Ll: Length of the distribution line l (km)

nc: Number of conductors available for installation

nl: Number of lines available for construction in the distribution system

nn: Number of nodes in the system

Nt: Years in the planning horizon

Sdk: Complex power demanded at node k by a load (kVA)

T : Number of hours in a year (hours)

Vl: Voltage drop across distribution line l (kV)

Vk: Voltage at node k (kV)

Vmax: Maximum voltage limit of node k (kV)

Vmin: Minimum voltage limit of node k (kV)

Xl: A binary variable that determines whether a distribution line l is constructed or not

Ylc: Binary variable entrusted with selecting the type C conductor to be installed in distribution line l

Z_c : Impedance of the conductor size c selected for line l (Ω/km)

Z_{cost} : Objective function associated with the system's annual operating costs (USD)

Z_{loss} : System energy loss costs (USD)

Z_{inv} : Investment costs of the conductors (USD)

C : Set containing the conductor sizes to be installed

L : Set containing the lines available for construction

N : Set containing the nodes in the distribution system

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Conflict of interest

None

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Ethics statement

None

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