ORIGINAL RESEARCH **Open access**

Numerical Simulation of DC Motor Speed Control Using Fractional-Order PID Controllers via Adams–Bashforth–Moulton Method

Ethan Wright[1], Chloe Bennett[1*], Jack Turner[2]

Abstract

A model regulating the speed of Direct Current (DC) motors has been established through the creation of various simulation models that utilize Proportional Integral Derivative (PID) controllers integrated with the fractional-order Adams–Bashforth–Moulton (ABM) approach. To gain more effective insights, a universal closed-loop framework for PID-type controllers was designed and implemented. The PID control architecture relies on a fundamental rule-set to oversee various environmental parameters. Managing mechanisms via Fractional-Order Controllers (FOC) in practical applications necessitates methods for constructing these controllers and tuning their parameters to ensure precise and accurate observation. Because traditional PID controllers are notoriously vulnerable to uncertainties stemming from an inexact understanding of system kinematics and dynamics, an Adaptive Fractional PID (AFPID) controller is proposed to leverage the inherent robustness of fractional-order control. While earlier research typically maintained static FPID controller parameters throughout the operation, this investigation dynamically updates these parameters online via a suitable adaptation mechanism to achieve superior outcomes. The resulting consistencies mark progress in comprehending the connection linking fractional calculus with chaotic phenomena. Observations indicate that P control dynamics enhance controller efficacy by expanding the available tuning parameters. Furthermore, initialization and execution times experienced a substantial reduction, dropping from 2.64 to 0.87 secs and 0.5 to 0.15 secs, respectively.

Keywords Direct current (DC) motors, Proportional Integral Derivative (PID), Fractional order Controllers (FOC), Adaptive Fractional PID (AFPID), Fractional calculus, Adams–Bashforth–Moulton (ABM) method

*Correspondence:
Chloe Bennett
chloe.bennett@leeds.ac.uk
[1] Department of Intelligent Grid Engineering, Faculty of Engineering, University of Leeds, Leeds, United Kingdom
[2] Department of Artificial Intelligence Applications, Faculty of Engineering, University of Sheffield, Sheffield, United Kingdom

Introduction

Within the domain of calculus, derivatives are a crucial subject of study for understanding a wide array of physical phenomena. Fundamentally, they illustrate the sensitivity to shifts in a function curve, commonly denoted as the slope or rate of change. The concept of the ‘derivative’ was initially introduced by Newton in 1666. He formulated this definition to conceptualize the physical meaning of instantaneous velocity, defined as speed coupled with direction [1]. To this day, scholars rely on this definition to characterize the intuitive geometric properties conveyed through the derivative. This is evident in mechanics, where distinct orders of displacement derivatives are established, such as the first derivative representing velocity and the second-order derivative denoting acceleration.

This is an open access article distributed under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License (CC BY-NC-SA 4.0), which permits non-commercial use, distribution, reproduction, and adaptation in any medium or format, provided that appropriate credit is given to the original author(s) and source, a link to the license is provided, and any changes made are indicated. Adapted material must be distributed under the same license.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

Furthermore, the principle of integration designates the area encompassed by a function curve. This concept was also primarily pioneered by Newton, establishing calculus as a comprehensive, independent field of study. It was subsequently refined over four centuries, eventually becoming recognized as ‘classical calculus’ [2].

Currently, numerous researchers and inventors strive to formulate models that capture the core traits of these phenomena using differential equations, whether ordinary, partial, or involving various integer-order variables. In many instances, traditional calculus proves inadequate for accurately depicting the complexities of the phenomenon. For example, in the mechanics of an ideal spring, the relationship between force and displacement during minor linear deformation associates force with the zeroth derivative with respect to displacement. Conversely, in ideal damping, force is strictly proportional to velocity, acting as the first derivative with respect to displacement. Recognizing this, scholars aim to mathematically model and simulate the nonlinear tendencies of mechanical structures using the innovative perspective of fractional calculus [3].

Investigations have evaluated COVID-19 treatment rates and eradication strategies via a mathematical perturbation approach [4]. The spatiotemporal transmission of COVID-19 was also scrutinized utilizing a fractional-order mathematical framework [5]. Analytical solitons were further explored by enhancing the homotopy perturbation technique applied to the fractional-order Korteweg–de Vries equation [6]. Nonconvex optimization strategies have been deployed to configure PI systems [7]. FO-PD control architectures were calibrated utilizing a fractional system subset [8]. Traditional PID performance regarding power system load frequency was elevated via particle swarm optimization paired with ANFIS [9]. Numerical simulations employing FABM methodologies were executed for FODEs [10]. IO frameworks were synthesized for IMC-PID filters [11]. AI-driven hybrid mechanisms were structured to address real-world challenges, including river water quality, IoT malware, and fuzzy-PID controls within electric battery vehicles [12-14]. A comprehensive examination of PID controllers featuring novel tuning techniques was conducted [15]. An adaptive architecture for robotic manipulation was established leveraging fractional PIDs [16]. PID configurations within AVR systems were optimized utilizing a particle swarm methodology [17]. The conventional PID controller was thoroughly detailed [18]. In-depth analyses were performed on inductive power systems integrated into electric vehicles [19]. A technical report detailing the calibration of fuzzy-PID frameworks was provided [20]. The continuous process programmability of logical controllers was comprehensively explained [21]. The function of formulating adaptive fuzzy PID controllers to manage BLDC motor speeds was detailed [22]. The automated generation process for governing power systems via ANFIS was simulated [23]. A fractional-order system incorporating dead time and 2-DoF was synthesized [24]. The robustness and stability of fractional PIs operating within time-delay systems were evaluated [25]. A nonlinear differential evolution algorithm was organized for interconnected power networks [26]. A self-calibrating PI and PID framework for control applications was introduced [27]. Inductive power transfer configurations were analyzed by refining PID controllers through a genetic algorithm [28]. Comprehensive insights into PI and PID systems, alongside their tuning mechanisms, were provided [29]. Mathematical financial models utilizing a Caputo fractional-order system were constructed to optimize demands and interest rates [30]. A COVID-19 mathematical framework was formulated employing Laplace Adomian decomposition [31]. The vaccination and transmission dynamics of diphtheria were mathematically modeled [32]. Calibrations achieving optimality in fuzzy and stochastic PIDs for networked architectures were performed [33]. Biomimetic approaches to optimization, control systems, and automation were investigated [34]. Existing literature regarding fractional PIDs was systematically reviewed [35]. Fractional-based controllers were utilized to oversee system chaos [36]. Recently, control engineering applications have increasingly leveraged frameworks founded on fractional nonlinear dynamics. To stabilize volatile open loops, a fractional-order controller was suggested [37]. The pros and cons of distinct practical digital executions for non-integer-order controllers in robotic arms were evaluated. Additionally, a fractional derivative controller is applied to mold an outer loop into a closed-form FO integrator [38].

Materials and Methods

This section encompasses fundamental preexisting theorems and definitions concerning fractional-order calculus, detailing the proposed expansion of half-order expressions into non-integral differentiation and integration operators.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

Fractional calculus preliminaries

As foundational calculus based on integer-order computations progressed, the underlying framework of fractional calculus simultaneously took shape. In 1695, L'Hospital and Leibniz explored the concept of half-order derivatives [15]. Fractional calculus is regarded as the expansion of differentiation and integration to encompass non-integer-order operators. Broadly, the lack of uniqueness regarding fractional-order derivatives has been extensively debated. A vast array of published literature and research corroborates this. Among the most frequently utilized are:

Definition of the grunwald–letnikov derivative

By expanding classical integral and derivative definitions utilizing fractional operators via a fractionalization algorithm, the Grunwald–Letnikov Derivative is formulated, depicted in **Eq. 1** as:

$$D^{\lambda}h(t) = \lim_{n\to 0} \frac{1}{n^{\lambda}} \sum_{m=0}^{\infty} (-1)^m \binom{\lambda}{m} h(t-mn)\ ,\ n \in N \qquad (1)$$

The binomial coefficient featured within this derivative is calculable using the Gamma function, shown in **Eq. 2** as:

$$\binom{\lambda}{m} = \frac{\lambda(\lambda-1)(\lambda-2)...(\lambda-m+1)}{m!} \qquad (2)$$

Because Aleksey Letnikov (1868) and Anton Karl Grunwald (1867) independently formulated and analyzed this derivative, it bears their names. It serves as the most foundational extension transitioning natural derivatives to fractional ones, originating directly from standard derivative operators.

Theorem Riemann Liouville Fractional Derivative (RLFD)

In 1847, Liouville alongside Riemann formulated this derivative, presented in **Eq. 3** as:

$$\begin{aligned} {}_{l}^{R}D_{u}^{\lambda}h(t) &= \left(\frac{d}{dt}\right)^{n}\left({}_{l}D_{u}^{-(n-\lambda)}\right)h(t) \\ &= \frac{1}{\Gamma(n-\lambda)}\left(\frac{d}{dt}\right)^{n}\int_{l}^{u}\frac{h(y)}{(t-y)^{\lambda-n+1}}dy\ , \end{aligned} \qquad (3)$$

where,
$\lambda > 0\,;\ n = [\lambda] + 1\,;\ u > l$
R – Riemann – –Liouville
l – lower bound
u – upper bound
λ – fractional – order

Featuring Γ (.) as the widely recognized gamma function, evaluated via:

Proof

This derivative is directly attainable by executing fractionalization on Cauchy's integral operator of order n, provided in **Eq. 4:**

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

$$ {}_{\lambda}D^{-n}h = \int_{\lambda}^{y_n}\int_{\lambda}^{y_{n-1}}\dots\int_{\lambda}^{y_1} h(y_0)\,dy_0\dots dy_{n-1}, n \in \mathrm{N} \quad (4) $$

Subsequently, applying the fractionalization algorithm to broaden this fractional expression, specifically as $n \to \lambda$ within **Eq. 5**

$$ {}_{\lambda}D^{-n}h = \frac{1}{(n-1)!}\int_{\lambda}^{y}(y-\tau)^{n-1}h(\tau)\,d\tau \quad (5) $$
$$ \tau - time\ function $$

Whenever $0 < \lambda < 1$, the RL operator simplifies the expression into **Eq. 6:**

$$ {}_{l}^{R}D_{u}^{\lambda}h(t) = \frac{1}{\Gamma(1-\lambda)}\frac{d}{dt}\int_{l}^{u}\frac{h(y)}{(t-y)^{\lambda}}dy \quad (6) $$

This corresponds to the simplified RLFD presented in **Eq. 3.**

Definition of the caputo derivative for fractional order

The Caputo derivative, applicable to fractional orders where n - 1 < λ < n, is given by **Eq. 7:**

$$ {}_{l}^{C}D_{u}^{\lambda}h(t) = \frac{1}{\Gamma(n-\lambda)}\int_{l}^{u}\frac{D^{n}h(\tau)}{(t-\tau)^{\lambda-n+1}}d\tau\ ,\ n-1<\lambda<n \quad (7) $$

This specific derivative is utilized to characterize highly complex models, particularly when the Riemann-Liouville Fractional Derivative (RLFD) proves inadequate for managing intricate system dynamics.

Remark

The outcomes derived from various fractional derivatives of integer order are generally equivalent; however, discrepancies may arise when dealing with non-integer orders.

Definition of Riemann–Liouville Fractional Integral (RLFI)

Assuming λ > 0 and $h \in L1([0, u], R)$ with $[0, u] \subset R+$, the Riemann-Liouville fractional integral of order λ for a given function h(t) is computed according to **Eq. 8:**

$$ I_{0+}^{\lambda} = \frac{1}{\Gamma(\lambda)}\int_{0}^{u}[(t-\tau)^{\lambda-1}g(\tau)]d\tau\ ,\ t>0 \quad (8) $$

Definition of caputo fractional integral

The Caputo-sense integral for the order n - 1 < λ < is expressed in **Eq. 9:**

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

$$ {}^{CD}D_{0+}^{\lambda} h(t) = \frac{1}{\Gamma(n-\lambda)} \int_0^u \frac{h^n(\tau)}{(t-\tau)^{\lambda-n+1}} d\tau \ , \ t > 0; \ n-1 < \lambda < n \quad (9) $$

Where (.) represents the standard gamma function, evaluated using **Eq. 10:**

$$ \Gamma(\lambda) = \int_0^{\infty} \tau^{\lambda-1} e^{-\tau} d\tau \quad (10) $$

Building upon these foundational definitions, we advance to Fractional-Order Dynamic Systems (FODS). These systems are evaluated utilizing the fractional differential equations shown in **Eq. 11:**

$$ \begin{aligned} &\varphi_n D^{\lambda_n} g(t) + \varphi_{n-1} D^{\lambda_{n-1}} g(t) + \cdots . + \varphi_0 D^{\lambda_0} g(t) \\ &\quad = \beta_p D^{\eta_p} v(t) + \beta_{p-1} D^{\eta_{p-1}} v(t) + \cdots . + \beta_0 D^{\eta_0} v(t) \\ &\textit{having} \\ &D^{\lambda} = {}_0D_t^{\lambda} \, ; D^{\eta} = {}_0D_t^{\eta} \\ &\varphi_k \ \forall \, k = 0, 1, 2, \ldots, n \, ; \beta_k \ \forall \, k = 0, 1, 2, \ldots, p \text{ are constants} \\ &\lambda_k \ \forall \, k = 0, 1, 2, \ldots, n; \eta_k \ \forall \, k = 0, 1, 2, \ldots, p \text{ are arbitrary real nos.} \end{aligned} \quad (11) $$

WLOG, it is established that $\lambda n > \lambda n\text{-}1 > \cdots . > \lambda 0$ & $\eta p > \eta p\text{-}1 > \cdots . > \eta 0$.

Subsequently, by employing discretization approximations for fractional-order integro-differentiators, the generalized discrete transfer function implemented within a controller is formulated in **Eq. 12:**

$$ \begin{aligned} &F(z) = \frac{\beta_p(\omega(z^{-1}))^{\eta_p} + \ldots . + \beta_0(\omega(z^{-1}))^{\eta_0}}{\varphi_n(\omega(z^{-1}))^{\lambda_n} + \ldots . + \varphi_0(\omega(z^{-1}))^{\lambda_0}} \, , \\ &\omega(z^{-1}) : \text{shift operator, } z^{-1} \text{ depicted as a function of complex variable, z} \end{aligned} \quad (12) $$

This particular shift operator functions as the discrete counterpart to the Laplace transform operator.

Mathematical Modelling of PID Controllers

Formulating a mathematical model of the control system is a prerequisite for controller design. Consequently, while developing these administrative controllers, the specific parameters of the FPID controller, namely λ and η, must be established. A gradient-based adaptive approach can be employed to calibrate these FPID parameters alongside their respective gains.

As previously documented in Bhardwaj and Bangia [13], three distinct variations of PID controllers were analyzed utilizing an ordinary differential equation-based state-space representation, defined in **Eq. 13:**

$$ \begin{aligned} &\frac{dj}{dt} = \alpha j + \beta i \, , \quad \alpha = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \, ; \ \beta = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \\ &\phi = \gamma j + \omega \, i, \quad \gamma = [1 \ \ 0] \, ; \ \omega = [0] \, ; \ i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &j = \begin{bmatrix} \phi \\ \frac{d\phi}{dt} \end{bmatrix} \end{aligned} \quad (13) $$

j(t): state-vectors; i(t): driving-current (input); ϕ(t): angular-displacements of the outputs

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

$$\alpha = \begin{matrix} & j_1 & j_2 \\ j_1 \\ j_2 \end{matrix} \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} ; \beta = \begin{matrix} & j_1 \\ i_1 \\ i_2 \end{matrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad (14)$$
$$\gamma = \phi_1 \begin{matrix} & j_1 & j_2 \end{matrix} \begin{bmatrix} 0 & 1 \end{bmatrix} ; \omega = \phi_1 \begin{matrix} & i_1 \end{matrix} [0]$$

Here, the state-space models comprise time-domain expressions for generalized fractional-order linear time-invariant (LTI) architectures, specified as:

$$\begin{aligned} &D^q x(t) = A\,x(t) + B\,v(t),\ x \in \mathrm{R}^n,\ v \in \mathrm{R}^r\ \&\ g \in \mathrm{R}^p \\ &g(t) = \gamma\,x(t) \\ &D^q = {}_0D_t^q,\ q - \text{fractional integer} - \text{order} \end{aligned} \quad (15)$$

x(t): state-vectors; v(t): input-vector(driving current); g(t): output-vectors;

This framework incorporates the following matrices:

A - $n \times n$ (*real/complex - valued*) *vector matrix*

B - $n \times r$ (*real/complex - valued*) *vector matrix*

γ - $p \times n$ (*real/complex - valued*) *vector matrix*

State-space representations are deduced from the differential equations that govern the system's dynamics.

Theorem

A fractional transfer function, expressed as $(s) = (s)/P\,(s)$, is considered stable if and only if the following criteria are met within the σ-plane:

$$\begin{aligned} &|\arg(\sigma)| > q\frac{\pi}{2},\ 0 < q < 1,\ \sigma \in \mathrm{C},\ P(\sigma) = 0, \\ &where\ \sigma := s^q. \end{aligned} \quad (16)$$

This principle is recognized as Matignon's Stability theorem.

Remark

The notable special cases include:

1. If $\sigma = 0$, a singular root of $P\,(s)$ is present, resulting in system instability.
2. If q=1, the theorem transitions into the traditional pole location theorem, indicating the absence of poles within the closed right-half of the complex plane.

Furthermore, stability theory dictates that a linear time-invariant (LTI) model achieves stability when its characteristic polynomial yields negative roots—specifically, possessing negative real parts that appear as complex conjugates. This positioning signifies their placement on the left half of the complex plane. Conversely, evaluating the stability of a fractional-order LTI framework requires a substantially different approach compared to integer models. It is noteworthy

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

that a stabilized fractional configuration may actually contain roots situated in the right half of the complex plane. The fundamental stability criterion required for the system to remain stable is detailed in **Eq. 17:**

$$|\arg(eig(A))| > q\frac{\pi}{2},\ 0 < q < 1 \quad (17)$$

Where eig(A) denotes the set of eigenvalues corresponding to matrix A.

In the current investigation, modeling for DC motor speed regulation was executed by formulating various simulation models reliant on fractional-order PID controllers. To achieve enhanced results, a simple closed-loop architecture for PID-style controllers was designed and implemented, as illustrated in **Figure 1**. Furthermore, the construction of the AFPID controllers was performed utilizing SIMULINK.

The governing continuous equation is discretized to form discrete-system fractional-order $PI^{\eta}D^{\lambda}$ controllers, which are implemented according to the Kp gains. This discrete representation of the DC motor is designated as the 'Plant system'. Generalized non-integer order Adaptive Fractional PID (AFPID(t)) controllers are modeled in **Eq. 18:**

$$PI^{\eta}D^{\lambda}(t) = K_p\left(\varepsilon(t) + \frac{1}{T_i}D^{-\eta}\varepsilon(t) + T_d D^{\lambda}\varepsilon(t)\right),\ \forall \eta, \lambda \in \mathrm{R}^{+}$$

where
η, λ – positive non – integer orders of integration and derivation terms
T_i – constant of differentiation
T_d – constant of integration
K_p – proportional gain
D – Riemann – –Liouville's fractional operator as :
$D^{-\eta}f(t) = \frac{1}{\Gamma(n)}\int_0^t f(y)(t-y)^{n-1}dy$
n – non – integer order; $\Gamma()$ – gammafunction
$\varepsilon(t)$ – error function, i.e., $\varepsilon(t) = t + \alpha\, Sin(t),\ \alpha \in \mathrm{R}$ (18)

This representation is frequently referred to as the time-domain formulation and is inherently dependent on t. The corresponding error function traces either a straight line or a curve, dictated by the instantaneous error magnitude at every computational increment.

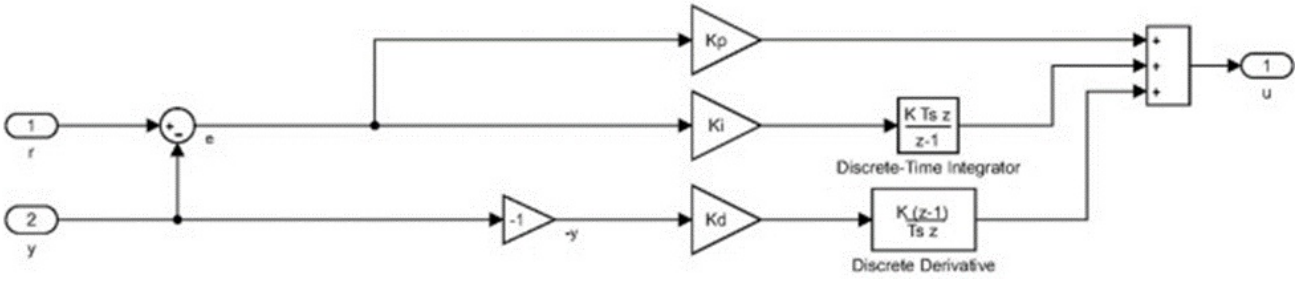


Figure 1. PID Controller designed in SIMULINK described in detail by Bhardwaj and Bangia [13].

Results and Discussion

The suggested control methodology was evaluated across various dynamic conditions. The simulation outcomes confirm the critical role of the fractional controller. Because fixed-step calculations form the foundation for assessing time

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

responses, the accuracy of the results inherently relies on the chosen step size. Nevertheless, validating a system's outputs must be prioritized as a crucial phase in their computation. Without an available analytical solution, an alternative viable method for verifying these results involves applying multiple step sizes to observe when consistent outputs occur. Within this research, an appropriately formulated rational discrete-time transfer function is presumed to maintain stability with a minimum phase configuration. Furthermore, to enhance the alignment of continuous frequency responses, the optimal approach is to compute discrete approximations featuring both poles and zeros located along the interval $z \in (-1, 1)$ on the z-plane. This strategy yields direct discretization approximations that possess all essential properties.

Consequently, the stability domain is graphically depicted in **Figure 2**. The comprehensive fluctuation in amplitude concerning time is illustrated in **Figure 3**. **Figure 4** presents the simulated temporal variations of the AFPID system parameters. The concurrent alterations in phase and magnitude relative to frequency are plotted in the Bode diagram in **Figure 5**. Moreover, the Bode diagram in **Figure 6** provides a comparison of phase and magnitude shifts between discrete and continuous-time approximations. **Figure 7** illustrates the performance assessment comparing the conventional model against the AFPID models to provide deeper insights into the system's adaptive characteristics. **Table 1** details the outcomes of two distinct PID models: one utilizing ordinary differentials and the other employing fractional-order differentials. **Table 2** contrasts the gain responses across the four synthesized PID controllers. **Table 3** outlines a literature review of prior investigations, contrasting their publication years, concise descriptions, applied parameters, and reported findings. This compilation facilitates a broader perspective and a clearer comprehension of recent scholarly progress relative to the current investigation.

Conclusion

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

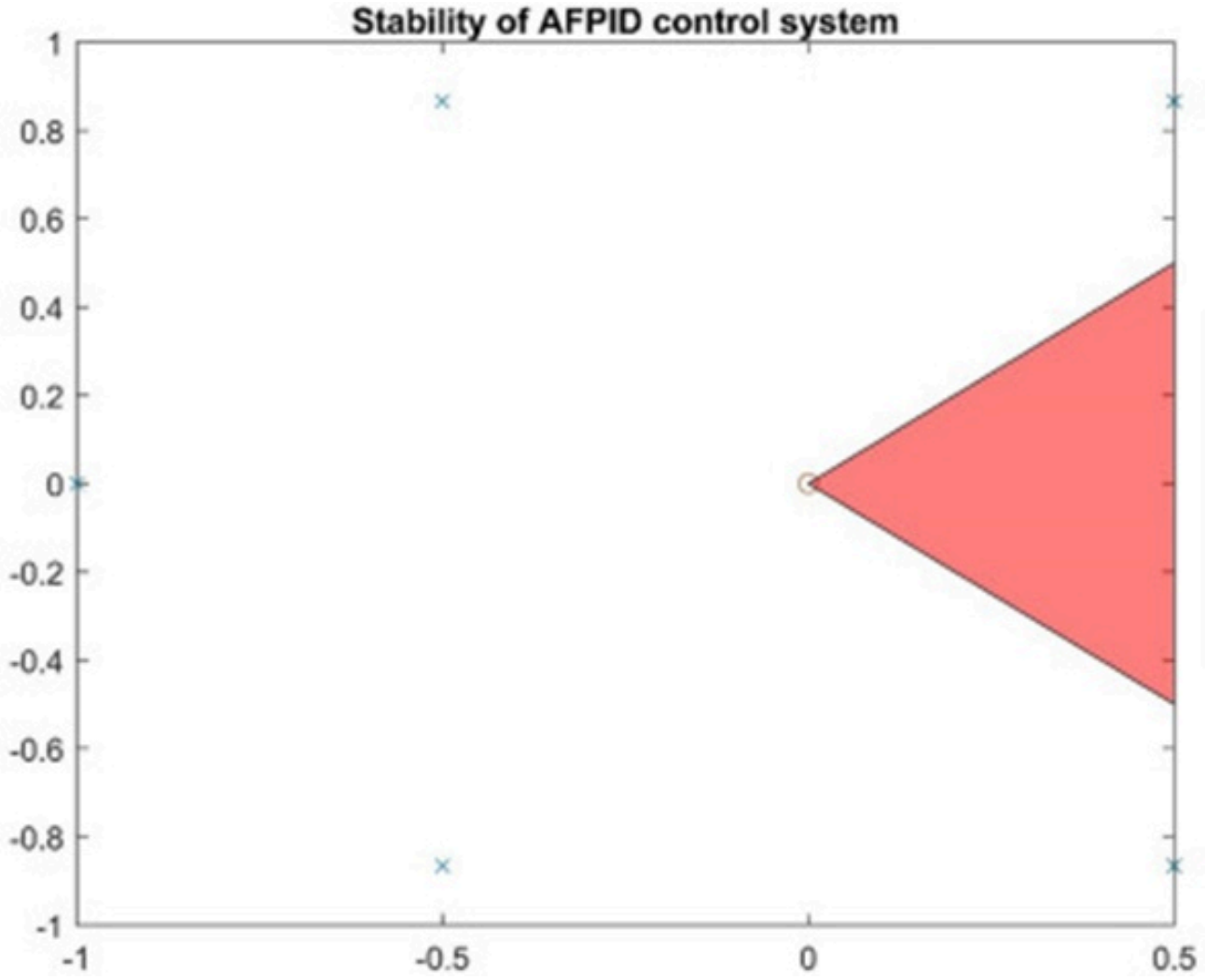


Figure 2. Stability region for the AFPID control system.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

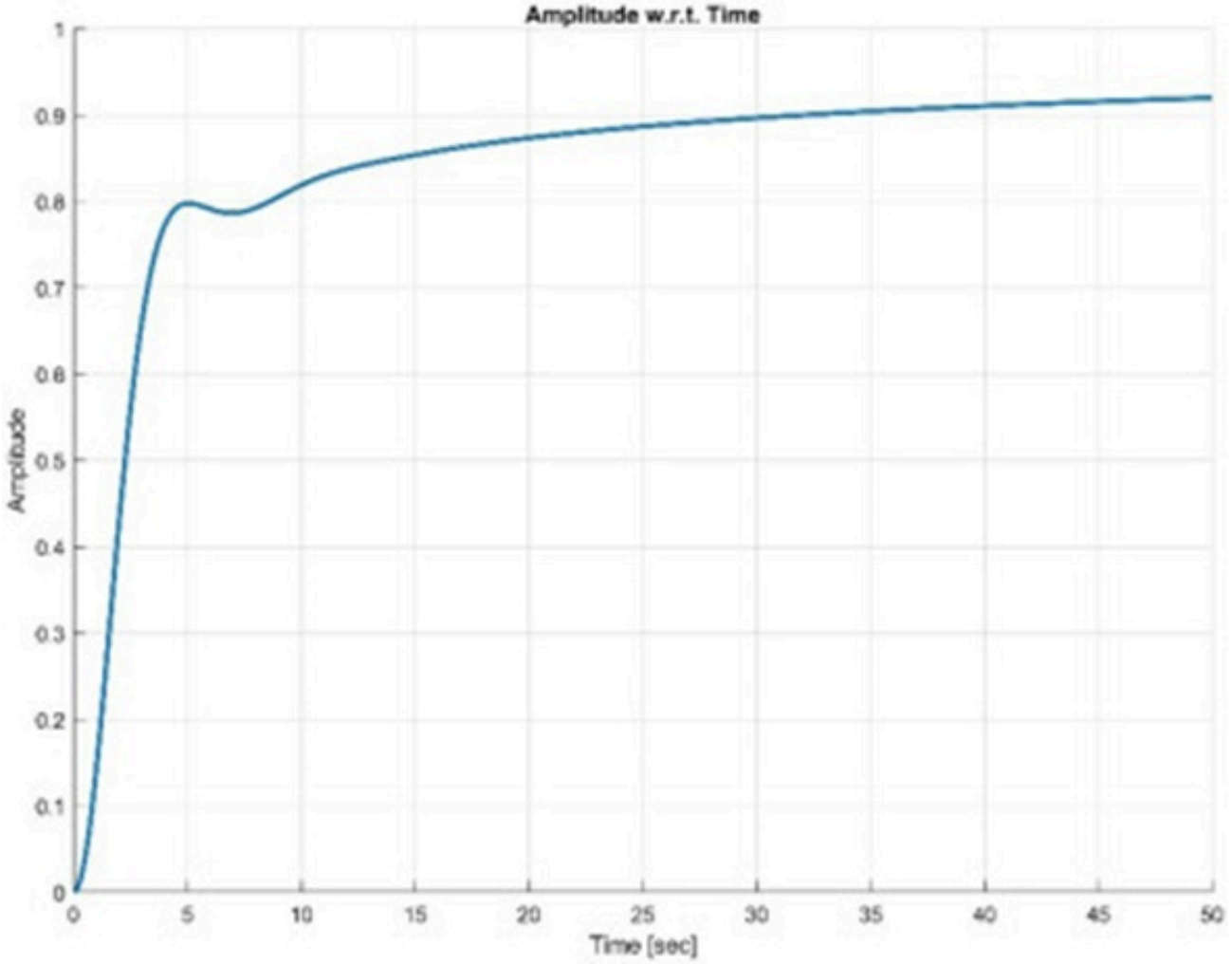


Figure 3. Amplitude w.r.t. time for the AFPID system.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

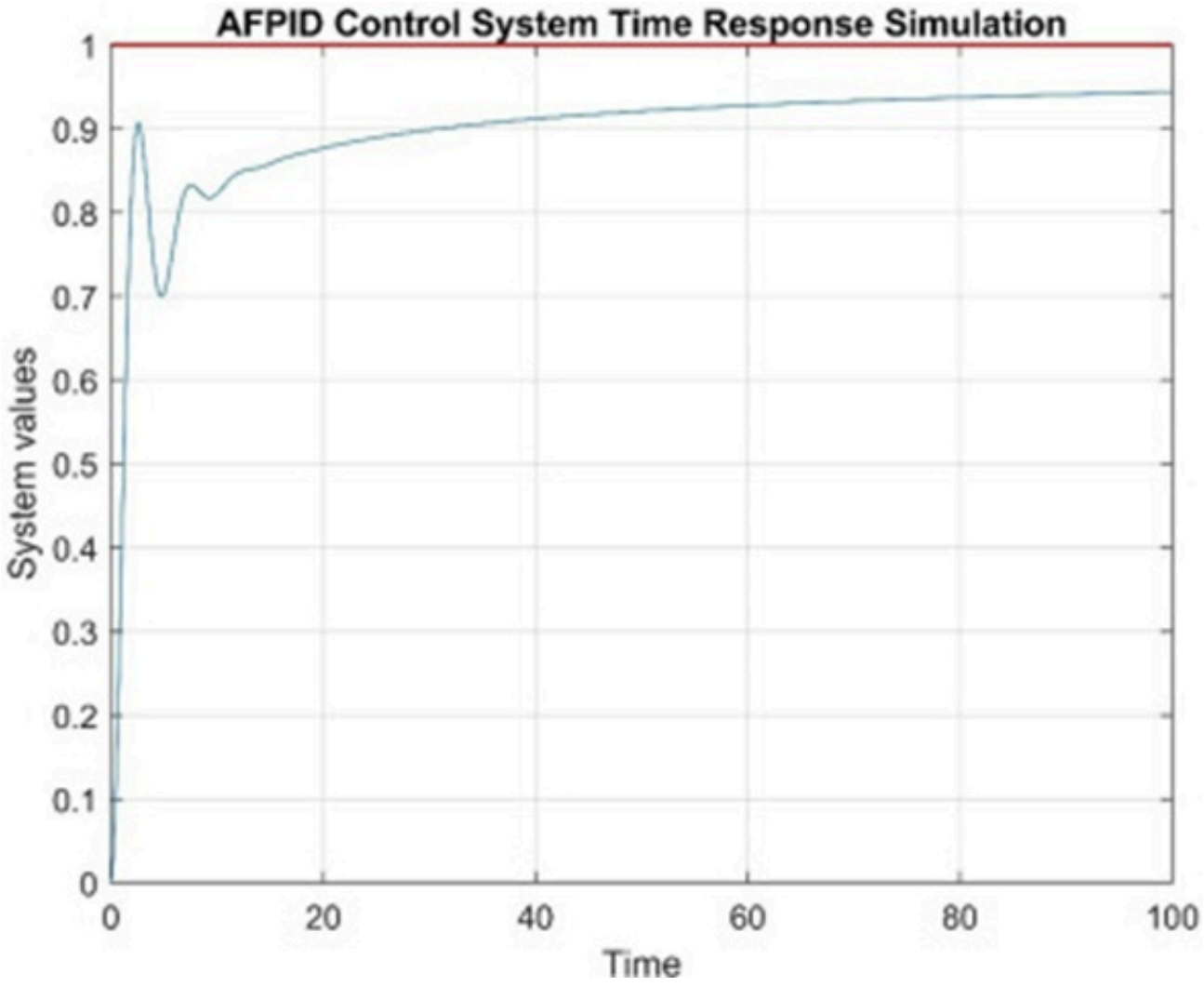


Figure 4. Simulation of AFPID system parameters' responses w.r.t. time.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

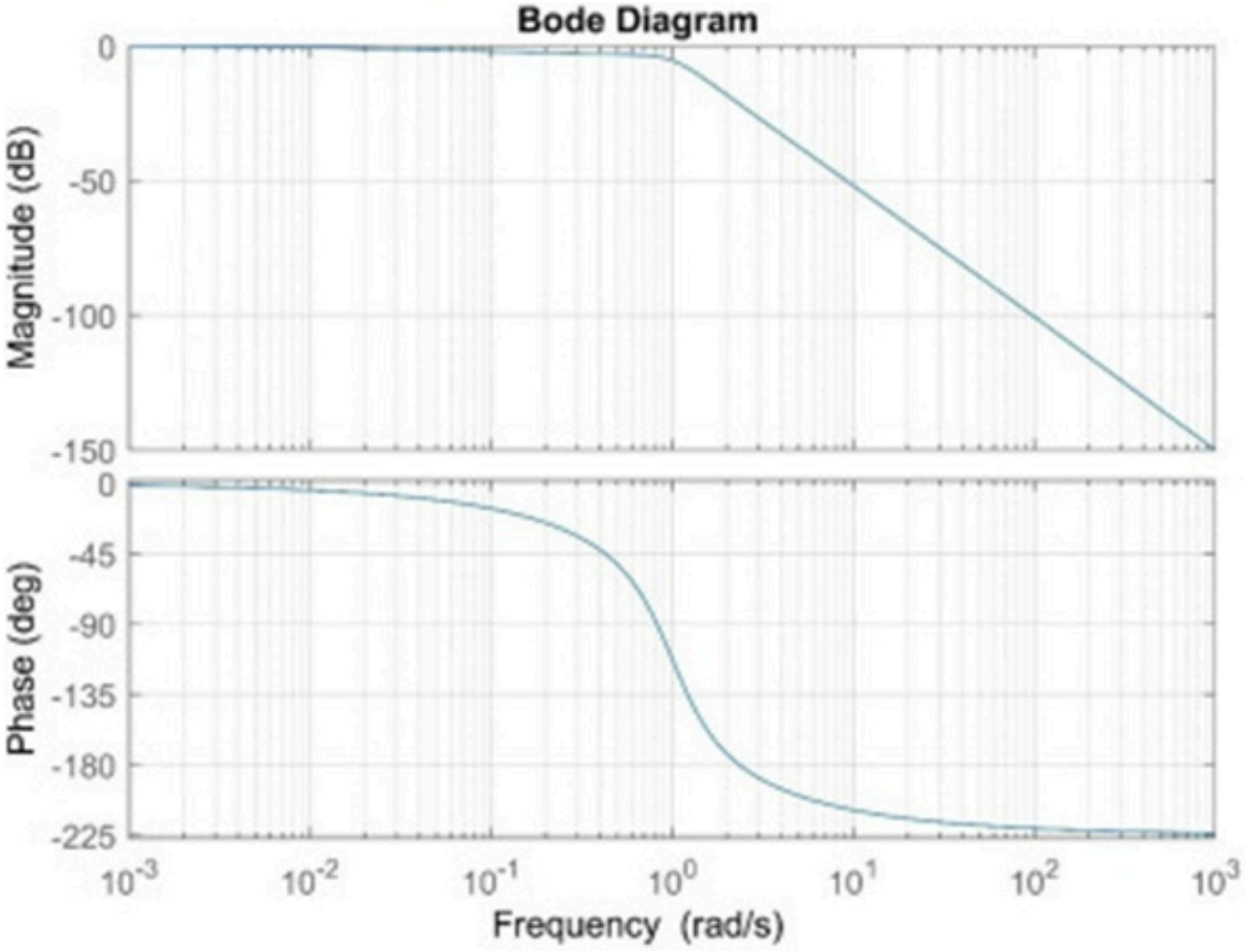


Figure 5. Bode Diagram showing the magnitude and phase changes w.r.t. frequency values.

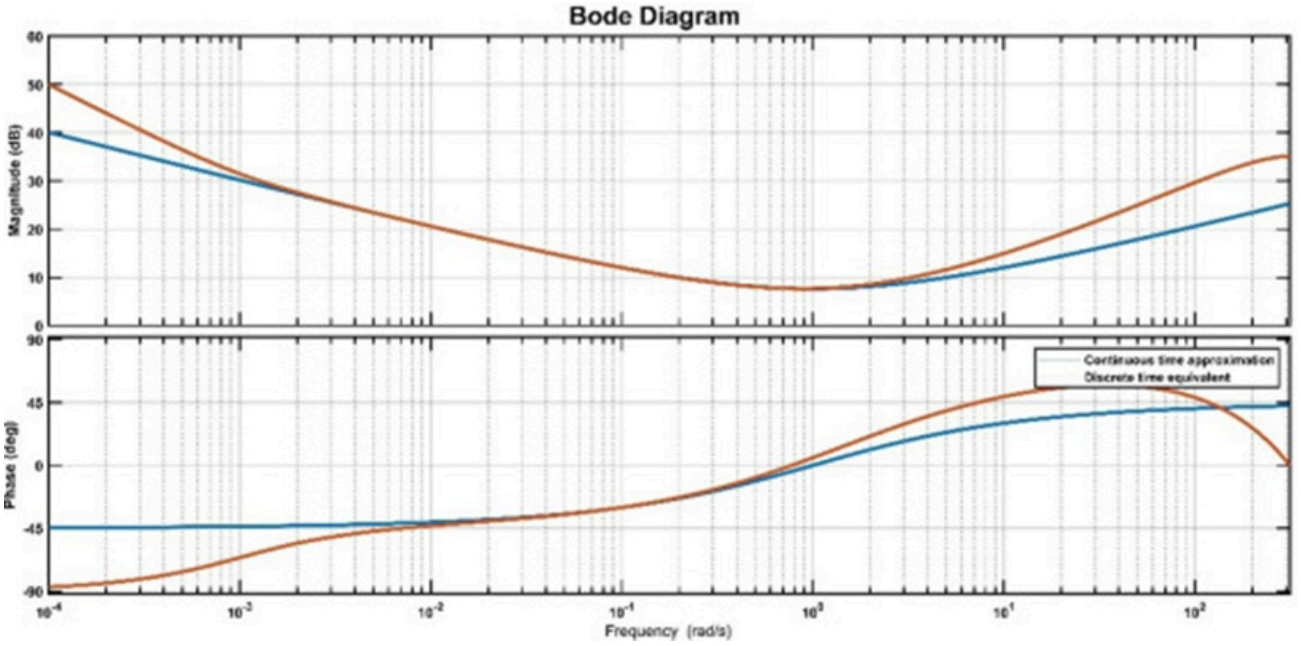


Figure 6. Bode diagram comparing the magnitude and phase changes of continuous and discrete-time approximations.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

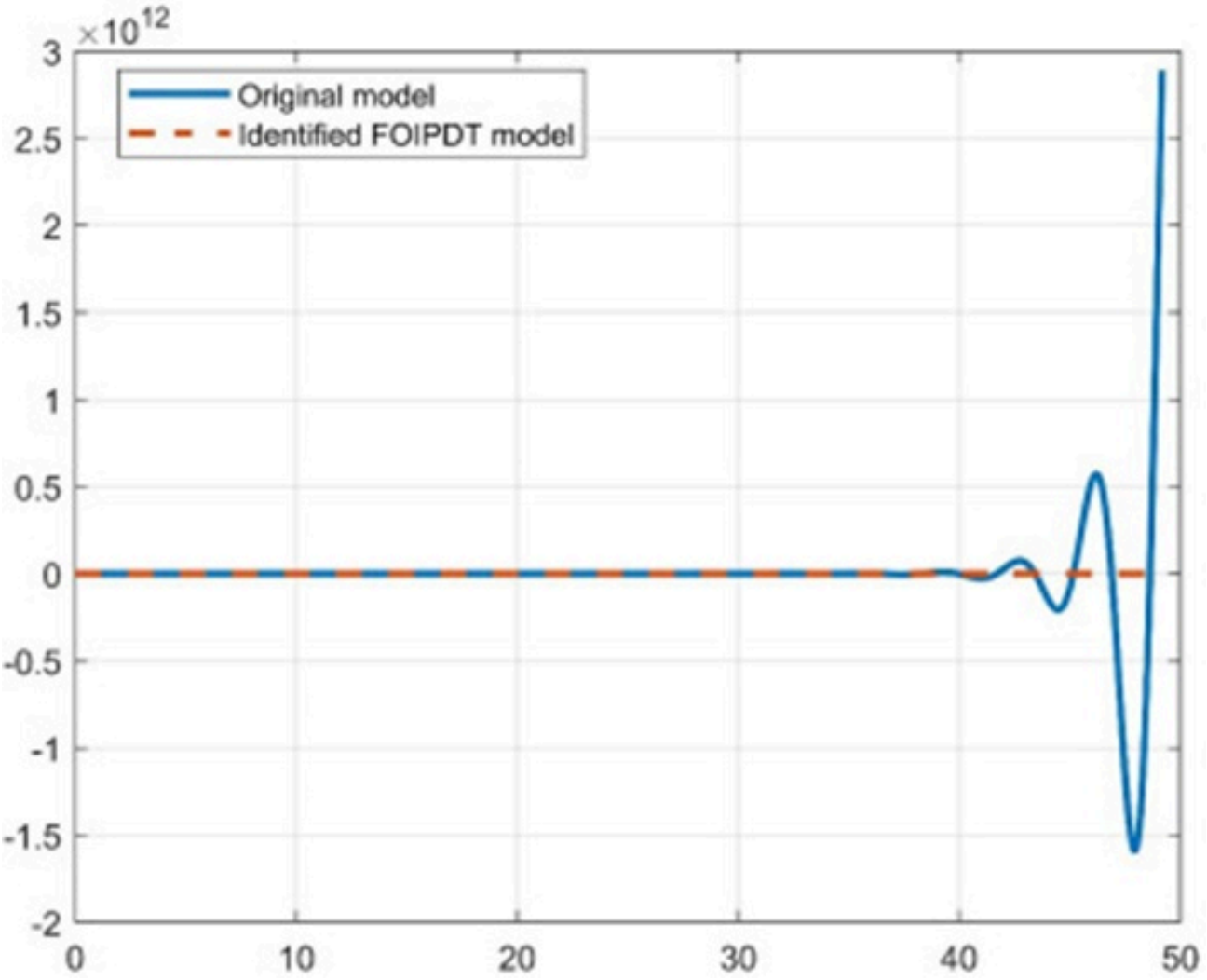


Figure 7. Performance evaluation of original model and AFPID models.

Table 1. Results of two different PID models.

Simulation name	ODE-based model [10]	AFPID model
Simulation mode	Normal	Normal
Start time	0	0
Stop time	12	12
Solver name	Variable-step	Variable-Step
Solver type	Variable step discrete	Variable Step Discrete
Initialization time (in seconds)	2.64	0.87
Execution time (in seconds)	0.48	0.15

Table 2. Comparison of gain responses from four types of controllers.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

Parameter	Speed of response	Stability	Accuracy
Conventional-PID	Response speed is improved compared to systems without a controller	Remains reasonably stable	Provides better accuracy than non-controller-based systems
Hybrid-Fuzzified-PID (linear control surface)	Faster response compared to conventional PID controllers	Steady-state error approaches zero	Shows improved accuracy relative to standard PID controllers
Hybrid-Fuzzified-PID with exhausted lookup tables (nonlinear control surface)	Exhibits a significantly higher response speed compared to the other models in this study	Steady-state error rapidly converges to zero	Performance difference is reduced compared to the linear fuzzy-PID model
Fractional ordered controllers	Response speed can be adjusted using parameters (\alpha, \beta) depending on system dynamics	Stability region can be analyzed through poles and zeros distributed along (z \in (-1, 1)) on the z-plane	Highest accuracy is achievable based on step size selection and proper tuning of fractional orders

Table 3. Comparison of various fractional and fuzzified PID controller studies.

Year	Author	Description	Parameters	Results
October, 2010	Delavari *et al*. [16]	Adaptive fractional PID controller developed for robotic applications	Optimization performed using a genetic algorithm	Numerical simulations demonstrate convergence behaviour of the proposed controller
August, 2015	Baskonus and Bulut [10]	Investigation of numerical solutions for fractional ordinary differential equations using the fractional Adams–Bashforth–Moulton method	Application of the fractional Adams–Bashforth–Moulton scheme on a two-dimensional surface	Both numerical and analytical solutions were obtained under appropriate parameter settings
September, 2019	Bhardwaj and Bangia [14]	Design of a fuzzy-enhanced PID controller for DC motor efficiency improvement in electric battery-powered vehicles	Development of a fuzzy-augmented PID controller for DC motor control	Model formulation analyzed in terms of stability and response speed characteristics
2024 (Present study)	Bhardwaj and Bangia	Simulation of a fractional-order control model for DC motor speed regulation	Fractional-order model developed and evaluated	System analyzed in terms of stability, phase behaviour, and amplitude variations

Findings from the fractional PID controller analysis demonstrate consistency, thereby advancing our understanding of the correlation bridging fractional calculus and stochastic behavior. Significant adaptability can be expected when pairing non-integer-order controllers with integer-order plants, including the capacity to adjust phase shifts or gains more effectively than traditional IO control setups. Because they require tuning fewer variables, the inherent flexibility of fractional-order

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

controller architectures has emerged as a crucial asset for developing highly efficient systems. Consequently, evidence indicates that fractional-order controllers can achieve a level of robustness comparable to that of high-order integer control models simply by calibrating a select few parameters. Fractional-order PID controllers currently present a highly precise approach that delivers greater industrial benefits than standard fractional-order controllers alone. Comprehensive numerical methodologies for the simulation of fractional-order PID frameworks have been established. It has been documented that, through meticulous fine-tuning, these controllers substantially improve the efficacy of system speed regulation. Theoretical principles suggest that, due to the presence of fractional-order differentiation and integration, these mechanisms inherently function as infinite-dimensional linearized filters. FOPID controllers are applicable across a broad spectrum of real-world scenarios. Reflecting upon the outcomes associated with integer-order PID mechanisms, it is highly probable that the AFPID configuration will achieve wider acceptance and demonstrate superior efficiency.

Acknowledgements

The authors are thankful to Sharda University and Guru Gobind Singh Indraprastha University for providing research facilities. No conflict of interest for publication of this article. No associated data, as the results were simulated using PID models.

Conflict of interest

None

Financial support

None

Ethics statement

None

Received: 20 Feb 2023 Revised: 28 Apr 2023 Accepted: 15 May 2023
Published online: 15 November 2024

Rights and permissions

Open Access The author(s) retain copyright. This article is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. It may be shared and adapted for non-commercial purposes with appropriate attribution, an indication of changes, and distribution of adaptations under the same license. Third-party material may be subject to separate terms identified in its credit line. View the license at https://creativecommons.org/licenses/by-nc-sa/4.0/.

References

1. Boyer CB, Merzbach UC. A history of mathematics. New York: John Wiley & Sons; 2011.

2. Dalir M, Bashour M. Applications of fractional calculus. Appl Math Sci. 2010;4(21):1021-32.

3. Magin RL. Fractional calculus in bioengineering. Redding: Begell House; 2006.

4. Adebisi AF, Olayiwola MO, Adediran IA, Adebayo KJ, Ogunleye RO, Akinwumi TO. A novel mathematical model and homotopy perturbation method analyzing the effects of saturated incidence and treatment rate on COVID-19 eradication. Iran J Sci. 2024;48:625-36.

5. Alaje AI, Olayiwola MO. A fractional-order mathematical model for examining the spatiotemporal spread of COVID-19 in the presence of vaccine distribution. Healthc Anal. 2023;4:100230.

6. Alaje AI, Olayiwola MO, Adedokun KA, Adeyemi OA, Bello RA, Ogunwale TA, et al. Modified homotopy perturbation method and its application to analytical solitons of fractional-order Korteweg–de Vries equation. Beni-Suef Univ J Basic Appl Sci. 2022;11:139.

7. Astrom KJ, Panagoulos H, Hagglund T. Design of PI controllers based on nonconvex optimization. Automatica. 1998;34(5):585-601.

8. Badri V, Tavazoei MS. On tuning fractional order proportional-derivative controllers for a class of fractional order systems. Automatica. 2013;49(7):2297-301.

9. Bahgaat NK, El-Sayed MI, Hassan MAM, Bendary FA. Load frequency control in power system via improving PID controller based on particle swarm optimization and ANFIS techniques. Int J Syst Dyn Appl. 2014;3(3):1-24.

10. Baskonus HM, Bulut H. On the numerical solutions of some fractional ordinary differential equations by fractional Adams–Bashforth-Moulton method. Open Math. 2015;13:547-56.

11. Bettayeb M, Mansouri R. IMC-pid-fractional-order-filter controllers design for integer order systems. ISA Trans. 2014;53(5):1620-8.

12. Bhardwaj R, Bangia A. Neuronal Brownian dynamics for salinity of river basins' water management. Neural Comput Appl. 2021;33:11923-36.

13. Bhardwaj R, Bangia A. Dynamical forensic inference for malware in IoT-based wireless transmissions. In: Sharma K, Makino M, Shrivastava G, Agarwal B, editors. Forensic investigations and risk management in mobile and wireless communications. Hershey: IGI Global; 2020. p. 51-79.

14. Bhardwaj R, Bangia A. Hybrid fuzzified-PID controller for non-linear control surfaces for DC motor to improve the efficiency of electric battery driven vehicles. Int J Recent Technol Eng. 2019;8(3):2561-8.

15. Cominos P, Munro N. PID controllers: Recent tuning methods and design to specification. Control Theory Appl. 2002;149(1):46-53.

16. Delavari H, Ghaderi R, Ranjbar A, HosseinNia SH, Momani S. Adaptive fractional PID controller for robot manipulator. In: Podlubny I, Vinagre Jara BM, Chen YQ, Feliu Batlle V, Tejado Balsera I, editors. Proceedings of the 4th IFAC Workshop on Fractional Differentiation and Its Applications (FDA'10); 2010 Oct 18-20; Badajoz, Spain. 2010. p. 18-20.

17. Gaing ZL. A particle swarm optimization approach for optimum design of PID controller in AVR system. IEEE Trans Energy Convers. 2004;19(2):384-91.

18. Goodwin GC, Graebe SF, Salgado ME. Classical PID control. In: Control system design. Upper Saddle River: Prentice Hall PTR; 2000.

19. Huh J, Lee SW, Lee WY, Cho GH, Rim CT. Narrow-width inductive power transfer system for online electrical vehicles. IEEE Trans Power Electron. 2011;26(12):3666-79.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press

20. Jantzen J. Tuning of fuzzy PID controllers: Technical Report. Lyngby: Dept. of Automation, Technical University of Denmark; 1999.

21. John W, Ronald W, Reis A. PID control of continuous processes. In: Programmable logic controllers. 4th ed. Upper Saddle River: Prentice Hall PTR; 1999.

22. Kandiban R, Arulmozhiyal R. Speed control of BLDC motor using adaptive fuzzy PID controller. Procedia Eng. 2012;38:308-13.

23. Khuntia SR, Panda S. Simulation study for automatic generation control of a multi-area power system by ANFIS approach. Appl Soft Comput. 2012;12:333-41.

24. Li M, Zhou P, Zhao Z, Zhang J. Two-degree of freedom fractional order-PID controllers design for fractional order processes with dead-time. ISA Trans. 2016;61:147-54.

25. Luo Y, Chen YQ. Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems. Automatica. 2012;8(9):2159-67.

26. Mohanty B, Panda S, Hota PK. Differential evolution algorithm based automatic generation control for interconnected power systems with non-linearity. Alex Eng J. 2014;53:537-52.

27. Mudi KR, Pal RN. A robust self-tuning scheme for PI-and PID type fuzzy controllers. IEEE Trans Fuzzy Syst. 1999;7(1):2-16.

28. Neath MJ, Swain AK, Madawala UK, Thrimawithana DJ. An optimal PID controller for a bidirectional inductive power transfer system using multiobjective genetic algorithm. IEEE Trans Power Electr. 2014;29(3):1523-31.

29. O'Dwyer A. Handbook of PI and PID controller tuning rules. 3rd ed. London: Imperial College Press; 2009.

30. Olayiwola MO, Alaje AI, Yunus AO. A Caputo fractional order financial mathematical model analyzing the impact of an adaptive minimum interest rate and maximum investment demand. Res Control Optim. 2024;14:100349.

31. Olayiwola MO, Alaje AI, Yunus AO, Adedokun KA, Bashiru KA. A mathematical modeling of COVID-19 treatment strategies utilizing the Laplace adomian decomposition method. Res Control Optim. 2024;14:100384.

32. Olayiwola MO, Alaje AI. Mathematical modelling of diphtheria transmission and vaccine efficacy using Nigeria. Model Earth Syst Environ. 2024;10:3941-67.

33. Pan I, Das S, Gupta A. Tuning of an optimal fuzzy PID controller with stochastic algorithms for networked control systems with random time delay. ISA Trans. 2011;50:28-36.

34. Passino KM. Biomimicry for optimization, control, and automation. London: Springer; 2004.

35. Shah P, Agashe S. Review of fractional PID controller. Mechatronics. 2016;38:29-41.

36. Tavazoei MS, Haeri M. Chaos control via a simple fractional-order controller. Phys Lett A. 2008;372:798-807.

37. Tepljakov A, Petlenkov E, Belikov J. FOMCON toolbox for modeling, design and implementation of fractional order control systems. In: Petráš I, editor. Applications in control. Vol. 6. Berlin: De Gruyter; 2019. p. 211-36.

38. Valério D, Costa JS. Digital implementation of non-integer control and its application to a two-link robotic arm. In: European Control Conference (ECC); 2003 Sep 1-4; Cambridge, UK. [place unknown]: [publisher unknown]; 2003.

© 2024 The Author(s). | Published by Institute of Smart Energy Systems Press