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Numerical Simulation of DC Motor Speed Control Using Adaptive Fractional $PI^{\eta}D^{\lambda}$ Controllers via Adams–Bashforth–Moulton Schemes

Nguyen Van Nam¹, Tran Thi Hoa^{1*}, Le Minh Duc²

Abstract

A model for speed control in Direct Current (DC) motors has been developed by creating various simulation models grounded in Proportional Integral Derivative (PID) controllers, utilizing the fractional-ordered Adams–Bashforth–Moulton (ABM) method. To achieve a deeper, more effective understanding, a universal closed-loop structure for PID-type controllers has been designed, along with its practical implementation. The PID control framework incorporates a set of rules necessary for tracking diverse environmental parameters. Implementing Fractional-order Control (FOC) in real-world applications requires approaches that enable controller design and parameter tuning for precise, reliable performance. Since conventional PID controllers are sensitive to uncertainties arising from imprecise knowledge of kinematics and dynamics, an Adaptive Fractional PID (AFPID) controller is proposed here to leverage the inherent robustness of fractional-order controllers. While earlier studies treated FPID controller parameters as fixed throughout the control process, this work introduces online parameter adaptation to achieve superior performance. The obtained results demonstrate consistency and mark an advancement in elucidating the connection between chaotic behavior and fractional calculus. It has been found that $PI^{\eta}D^{\lambda}$ control configurations can significantly enhance controller effectiveness by providing additional tuning parameters. Moreover, initialization and execution times have been markedly reduced, dropping from 2.64 to 0.87 seconds and from 0.5 to 0.15 seconds, respectively.

Keywords Direct current (DC) motors, Proportional Integral Derivative (PID), Fractional order Controllers (FOC), Adaptive Fractional PID (AFPID), Fractional calculus, Adams–Bashforth–Moulton (ABM) method

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Introduction

In calculus, derivatives are a fundamental tool for interpreting a wide range of physical phenomena. At its core, the concept illustrates a function's pronounced sensitivity to small variations, commonly expressed as the rate of change, or slope, of the curve. The idea of the 'derivative' was initially introduced by Newton in 1666. He formulated it to mathematically capture the physical meaning of instantaneous velocity, defined as the speed of an object at a given instant [1]. To this day, this definition remains widely used by researchers to explain the intuitive geometric interpretation of derivatives. In mechanics, for instance, successive orders of differentiation applied to displacement yield well-known quantities: the first derivative corresponds to velocity, while the second derivative corresponds to acceleration. Similarly, integration is defined as the area under a function's curve. This concept, too, was largely pioneered by Newton,

establishing calculus as a comprehensive discipline. Over more than four centuries, it evolved and became widely recognized as 'classical calculus' [2].

Today, numerous researchers and engineers strive to develop models that encapsulate the essential features of various phenomena using differential equations—whether ordinary, partial, or based on integer-order derivatives. However, in many situations, traditional classical calculus proves inadequate for accurately representing the intricate nature of these phenomena. Consider, for example, an ideal spring under small linear deformation, where force relates directly to displacement (the zeroth derivative with respect to displacement). In contrast, for an ideal damper, force is strongly proportional to velocity (the first derivative with respect to deformation). With this perspective in view, researchers are increasingly turning to the innovative framework of fractional calculus to model and simulate the nonlinear dynamics of mechanical systems [3].

The effects and treatment rates for COVID-19 eradication were analyzed using a mathematical model based on the perturbation method [4]. The spatiotemporal spread of COVID-19 was investigated through a fractional-order mathematical model [5]. An improved homotopy perturbation method was applied to obtain analytical soliton solutions for the fractional-order Korteweg–de Vries equation [6]. A nonconvex optimization technique was employed to design the PI system [7]. Fractional-order PD control systems were tuned using subsets of fractional systems [8]. The classical PID controller was enhanced for load-frequency control in power systems using particle swarm optimization combined with ANFIS [9]. Numerical simulations of fractional-order differential equations (FODEs) were performed using FABM methods [10]. Internal model control (IMC)-PID filters were designed for integer-order (IO) systems [11]. Artificial intelligence-based hybrid models were developed to address real-world problems, including river water quality modeling, malware detection in IoT networks, and fuzzy-PID control systems for electric battery vehicles [12–14]. A comprehensive study of PID controllers featuring new and improved tuning approaches was conducted [15]. An adaptive fractional PID system was constructed to support robot manipulation tasks [16]. PID controller design for automatic voltage regulator (AVR) systems was optimized using a particle swarm approach [17]. The fundamentals of classical PID controllers were explained [18]. A detailed analysis of inductive power transfer systems applied in electric vehicles was presented [19]. Technical aspects of tuning fuzzy-PID controllers were reported [20]. The programmability of logical controllers was described by treating them as continuous processes [21]. The development and role of adaptive fuzzy PID controllers in regulating the speed of BLDC motors were discussed [22]. ANFIS-based auto-generation was simulated for power system control [23]. A fractional-order system incorporating dead time and featuring two degrees of freedom (2-DoF) was designed [24]. The stability and robustness properties of fractional-order PI controllers in time-delay systems were examined [25]. A differential evolution algorithm that accounts for nonlinearity was developed for interconnected power systems [26]. Self-tuning PI and PID mechanisms for controller applications were introduced [27]. Inductive power transfer configurations were analyzed by optimizing PID controllers through genetic algorithms [28]. O'Dwyer [29] provided a thorough explanation of PI and PID controllers, including their tuning mechanisms. A financial mathematical model based on the Caputo fractional-order system was developed to optimize interest rates and demand [30]. A mathematical model of COVID-19 was constructed using the Laplace Adomian decomposition method [31]. The transmission dynamics and vaccination strategies for diphtheria were mathematically modeled [32]. PID controllers were tuned to achieve optimality in networked systems through fuzzification and stochastic approaches [33]. Biomimicry principles were explored in the context of optimization, control systems, and automation [34]. A literature review on fractional-order PID controllers was presented [35]. Fractional-based controllers were utilized to suppress chaos in dynamical systems [36]. In recent years, control engineering applications have increasingly adopted systems grounded in fractional nonlinear dynamics. A fractional-order controller was proposed to stabilize unstable open-loop systems [37]. The benefits and limitations of different digital implementations for non-integer-order controllers in robotic arms were evaluated. A fractional derivative controller was applied to configure an outer loop as a closed-form fractional-order integrator [38].

Materials and Methods

This section presents foundational definitions and theorems of fractional-order calculus, along with proposed generalizations that extend half-order derivatives and integrals to non-integer operators.

Fractional calculus preliminaries

Alongside the advancement of classical integer-order calculus, the conceptual framework of fractional calculus also emerged. In 1695, Leibniz and L'Hôpital explored the idea of half-order derivatives [15]. Fractional calculus is generally regarded as the extension of integration and differentiation to non-integer orders. The non-uniqueness of fractional-order derivatives has been widely discussed and is supported by extensive research and publications.

Definition: grünwald–letnikov derivative

By extending ordinary derivatives and integrals via a fractionalization procedure, the Grünwald–Letnikov derivative is obtained, expressed in Eq. 1 as:

$$D^\lambda h(t) = \lim_{n \rightarrow 0} \frac{1}{n^\lambda} \sum_{m=0}^{\infty} (-1)^m \binom{\lambda}{m} h(t - mn), \quad n \in \mathbb{N} \quad (1)$$

The binomial coefficients appearing in this derivative are calculated using the Gamma function, as shown in Eq. 2:

$$\binom{\lambda}{m} = \frac{\lambda(\lambda-1)(\lambda-2)\dots(\lambda-m+1)}{m!} \quad (2)$$

This derivative, developed and discussed by Anton Karl Grünwald (1867) and Aleksey Letnikov (1868), is named after them. It represents the most straightforward and natural generalization of the classical derivative to fractional orders, directly derivable from the standard differentiation operator.

Theorem: Riemann–Liouville Fractional Derivative (RLFD)

The Riemann–Liouville fractional derivative, formulated by Riemann and Liouville in 1847, is given in Eq. 3 as:

$$\begin{aligned} {}^R D_u^\lambda h(t) &= \left(\frac{d}{dt}\right)^n \left({}_l D_u^{-(n-\lambda)}\right) h(t) \\ &= \frac{1}{\Gamma(n-\lambda)} \left(\frac{d}{dt}\right)^n \int_l^u \frac{h(y)}{(t-y)^{\lambda-n+1}} dy, \end{aligned} \quad (3)$$

where,

$$\lambda > 0; n = [\lambda] + 1; u > l \quad (3)$$

R – Riemann – Liouville

l – lower bound

u – upper bound

λ – fractional – order

having $\Gamma(\cdot)$ as the well-known gamma function computed through

Proof

This derivative can be directly obtained by applying the fractionalization procedure to the Cauchy integral formula of integer order n , expressed as in [Eq. 4](#):

$${}_{{}_\lambda}D^{-n}h = \int_{{}_\lambda}^{y_n} \int_{{}_\lambda}^{y_{n-1}} \dots \int_{{}_\lambda}^{y_1} h(y_0) dy_0 \dots dy_{n-1}, n \in \mathbb{N} \quad (4)$$

Subsequently, by extending this expression through fractionalization — that is, replacing n with λ — the formulation becomes as shown in [Eq. 5](#):

$${}_{{}_\lambda}D^{-n}h = \frac{1}{(n-1)!} \int_{{}_\lambda}^y (y-\tau)^{n-1} h(\tau) d\tau \quad (5)$$

τ – time function

In the particular case where $0 < \lambda < 1$, the Riemann–Liouville operator simplifies to the form given in [Eq. 6](#):

$${}_l^R D_u^\lambda h(t) = \frac{1}{\Gamma(1-\lambda)} \frac{d}{dt} \int_l^u \frac{h(y)}{(t-y)^\lambda} dy \quad (6)$$

This simplified expression corresponds exactly to the reduced Riemann–Liouville fractional derivative presented earlier in [Eq. 3](#).

Definition: caputo fractional derivative

The Caputo derivative of fractional order is defined for $n-1 < \lambda < n$ as follows in [Eq. 7](#):

$${}_l^{CD} D_u^\lambda h(t) = \frac{1}{\Gamma(n-\lambda)} \int_l^u \frac{D^n h(\tau)}{(t-\tau)^{\lambda-n+1}} d\tau, n-1 < \lambda < n \quad (7)$$

This derivative is particularly suitable for modeling systems that exhibit complex behavior, as the Riemann–Liouville fractional derivative is often inadequate for such intricate dynamics.

Remark

When the fractional order is an integer, the results produced by different fractional derivative definitions are essentially identical; however, they generally differ when the order is non-integer.

Definition: Riemann–Liouville Fractional Integral (RLFI)

For $\lambda > 0$ and $h \in L^1([0, u], \mathbb{R})$ with $[0, u] \subset \mathbb{R}^+$, the Riemann–Liouville fractional integral of order λ applied to the function $h(t)$ is given by [Eq. 8](#):

$$I_{0+}^\lambda = \frac{1}{\Gamma(\lambda)} \int_0^u \left[(t-\tau)^{\lambda-1} g(\tau) \right] d\tau, t > 0 \quad (8)$$

Definition: caputo fractional integral

The Caputo fractional integral of order $n - 1 < \lambda < n$ is defined as in [Eq. 9](#):

$${}^{CD}D_{0^+}^{\lambda} h(t) = \frac{1}{\Gamma(n-\lambda)} \int_0^t \frac{h^u(\tau)}{(t-\tau)^{\lambda-n+1}} d\tau, \quad t > 0; \quad n - 1 < \lambda < n \quad (9)$$

Where $(.)$ represents the standard Gamma function, computed according to [Eq. 10](#):

$$\Gamma(\lambda) = \int_0^{\infty} \tau^{\lambda-1} e^{-\tau} d\tau \quad (10)$$

Building upon these foundational definitions, attention now turns to Fractional-Order Dynamic Systems (FODS). These systems are governed by fractional differential equations of the general form shown in [Eq. 11](#):

$$\begin{aligned} \varphi_n D^{\lambda_n} g(t) + \varphi_{n-1} D^{\lambda_{n-1}} g(t) + \dots + \varphi_0 D^{\lambda_0} g(t) \\ = \beta_p D^{\eta_p} v(t) + \beta_{p-1} D^{\eta_{p-1}} v(t) + \dots + \beta_0 D^{\eta_0} v(t) \end{aligned} \quad (11)$$

Without loss of generality, it is assumed that $\lambda_n > \lambda_{n-1} > \dots > \lambda_0$ & $\eta_p > \eta_{p-1} > \dots > \eta_0$.

To enable practical implementation, approximations are introduced for discretizing the fractional-order integro-differential operators. The resulting general expression for the discrete transfer function employed in controllers is presented in [Eq. 12](#):

$$F(z) = \frac{\beta_p (\omega(z^{-1}))^{\eta_p} + \dots + \beta_0 (\omega(z^{-1}))^{\eta_0}}{\varphi_n (\omega(z^{-1}))^{\lambda_n} + \dots + \varphi_0 (\omega(z^{-1}))^{\lambda_0}}, \quad (12)$$

(z^{-1}) : shift operator, where z^{-1} is expressed as a function of the complex variable z

This shift operator serves as the discrete-time counterpart to the continuous-time Laplace transform operator.

Mathematical Modeling of PID Controllers

To enable effective controller design, a precise mathematical representation of the control system is essential. Accordingly, when developing adaptive controllers, the fractional parameters of the FPID controller—namely λ and η —along with the associated gains, must be appropriately determined. An adaptive tuning approach grounded in the gradient method can be employed to adjust these fractional parameters and gains of the FPID controller.

As previously reported by Bhardwaj and Bangia [\[13\]](#), three categories of PID controllers have been analyzed through state-space representations derived from ordinary differential equations, expressed as in [Eq. 13](#):

$$\begin{aligned} \frac{dj}{dt} &= \alpha j + \beta i, \quad \alpha = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}; \quad \beta = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \\ \phi &= \gamma j + \omega i, \quad \gamma = [1 \ 0]; \quad \omega = [0]; \quad i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ j &= \begin{bmatrix} \phi \\ \frac{d\phi}{dt} \end{bmatrix} \end{aligned} \quad (13)$$

$j(t)$: state vectors; $i(t)$: driving current (input); $\phi(t)$: angular displacements of the outputs

$$\begin{aligned} \alpha &= \begin{matrix} j_1 & j_2 \\ j_1 & j_2 \end{matrix} \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix}; \quad \beta = \begin{matrix} j_1 \\ i_1 \\ i_2 \end{matrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} \\ \gamma &= \begin{matrix} j_1 & j_2 \\ i_1 \end{matrix} \begin{bmatrix} 0 & 1 \end{bmatrix}; \quad \omega = \begin{matrix} i_1 \end{matrix} [0] \end{aligned} \quad (14)$$

In the present context, the state-space formulations provide time-domain descriptions of fractional-order linear time-invariant (LTI) systems in their general form, given as:

$$\begin{aligned} D^q x(t) &= A x(t) + B v(t), \quad x \in \mathbb{R}^n, \quad v \in \mathbb{R}^r \quad \& \quad g \in \mathbb{R}^p \\ g(t) &= \gamma x(t) \\ D^q &= {}_0 D_t^q, \quad q - \text{fractional integer - order} \end{aligned} \quad (15)$$

$x(t)$: state vectors; $v(t)$: input vector (driving current); $g(t)$: output vectors

The following matrices characterize this model:

A – $n \times n$ (real or complex-valued) vector matrix

B – $n \times r$ (real or complex-valued) vector matrix

Γ – $p \times n$ (real or complex-valued) vector matrix

State-space models are obtained by converting differential equations that capture the system's dynamic behavior.

Theorem. A fractional transfer function $G(s) = Z(s)/P(s)$ is considered stable if and only if the following conditions hold with respect to the σ -plane:

$$\begin{aligned} |\arg(\sigma)| &> q \frac{\pi}{2}, \quad 0 < q < 1, \quad \sigma \in \mathbb{C}, \quad P(\sigma) = 0, \\ \text{where } \sigma &:= s^q. \end{aligned} \quad (16)$$

This result is commonly referred to as Matignon's stability theorem.

Remark. The theorem yields the following notable special cases:

1. When $\sigma = 0$, a single root of $G(s)$ lies on the imaginary axis, rendering the system unstable.
2. When $q = 1$, the theorem reduces to the classical pole-location criterion, which requires that no poles exist in the right-half of the closed complex plane.

Furthermore, stability theory indicates that a linear time-invariant (LTI) system is stabilized when the roots of its characteristic polynomial have negative real parts (either real negatives or complex conjugate pairs with negative real parts), placing them in the left half-plane. For integer-order LTI systems, this condition is standard; however, stability assessment for fractional-order LTI systems requires a distinct approach. Notably, in a stable fractional-order system, roots may lie in the right half-plane under certain conditions. The fundamental stability criterion for such systems is expressed in [Eq. 17](#) as follows:

$$|\arg(\text{eig}(A))| > q\frac{\pi}{2}, \quad 0 < q < 1 \quad (17)$$

Where $\text{eig}(A)$ denotes the set of eigenvalues of the matrix A .

In this work, the speed control of DC motors has been modeled using multiple simulation models with fractional-order PID controllers. To improve performance, a streamlined closed-loop configuration for PID-type controllers has been formulated and implemented, as illustrated in [Figure 1](#). Additionally, the design of Adaptive Fractional PID (AFPID) controllers has been implemented in MATLAB/SIMULINK.

The continuous-time equation is discretized to obtain the discrete-time fractional-order $PI^\eta D^\lambda$ controller, which is applied according to the proportional gain K_p . The discrete representation of the DC motor is referred to as the 'Plant system'. The generalized form of the adaptive fractional-order PID controller (AFPID(t)) with non-integer orders can be expressed as in [Eq. 18](#):

$$PI^\eta D^\lambda(t) = K_p \left(\varepsilon(t) + \frac{1}{T_i} D^{-\eta} \varepsilon(t) + T_d D^\lambda \varepsilon(t) \right), \quad \forall \eta, \lambda \in \mathbb{R}^+ \quad (18)$$

Where:

λ – positive non-integer orders of the integration and derivation terms

T_i – differentiation time constant

T_d – integration time constant

K_p – proportional gain

D – Riemann–Liouville fractional operator, defined as:

$$D^{-\eta} f(t) = \frac{1}{\Gamma(\eta)} \int_0^t f(y)(t-y)^{\eta-1} dy$$

η – non-integer order; $\Gamma(\cdot)$ – gamma function

$\varepsilon(t)$ – error function, i.e., $\varepsilon(t) = r(t) + \alpha \sin(t)$, $\alpha \in \mathbb{R}$

This formulation is also referred to as the time-domain expression, expressed in terms of t . The error function generates either a curved or a linear trajectory, depending on the instantaneous error at each computation step.

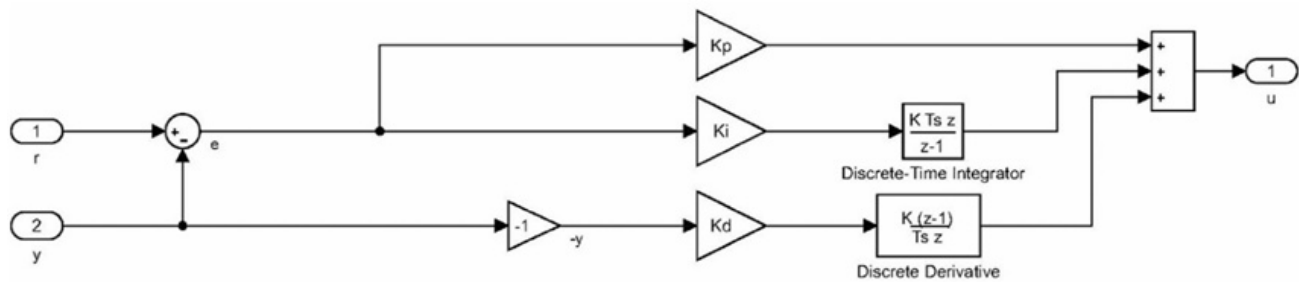


Figure 1. PID Controller implemented in SIMULINK, described in detail by Bhardwaj and Bangia [13].

Simulation Results and Discussion

The proposed control strategy was evaluated across a range of dynamic behaviors. The simulation outcomes confirm the effectiveness and importance of the fractional-order controller. Since time-response evaluations rely on fixed-step computations, the accuracy of the results is directly influenced by the chosen step size. Nonetheless, validating system outputs remains a critical procedure. In the absence of an analytical closed-form solution, a reliable approach to verification is to test multiple step sizes and confirm consistency across the results.

In this work, a suitably constructed rational discrete-time transfer function is assumed to be stable and minimum-phase. Moreover, to approximate continuous-frequency responses, the most effective strategy is to compute discrete approximations that position poles and zeros on the segment $z \in (-1,1)$ in the z -plane. This method yields direct discretization approximations that preserve all essential properties. Accordingly, the stability region is shown graphically in **Figure 2**. The complete variation in amplitude over time is presented in **Figure 3**. **Figure 4** illustrates the time evolution of the AFPID system parameters. Magnitude and phase variations with respect to frequency are jointly depicted in the Bode plot shown in **Figure 5**. Additionally, **Figure 6** provides a Bode diagram comparing magnitude and phase responses between continuous-time and discrete-time approximations. **Figure 7** compares the original model with the AFPID models, providing clearer insight into their adaptive characteristics.

Table 1 summarizes the results obtained from two distinct PID models—one based on ordinary differential equations and the other on fractional-order differentials.

Table 2 compares the gain responses of four different PID controller designs.

Table 3 presents a literature comparison of prior studies, including the year of publication, brief description, key parameters considered, and main findings reported in each work. This overview facilitates a comprehensive understanding of recent advancements in the field and positions the current study within that context.

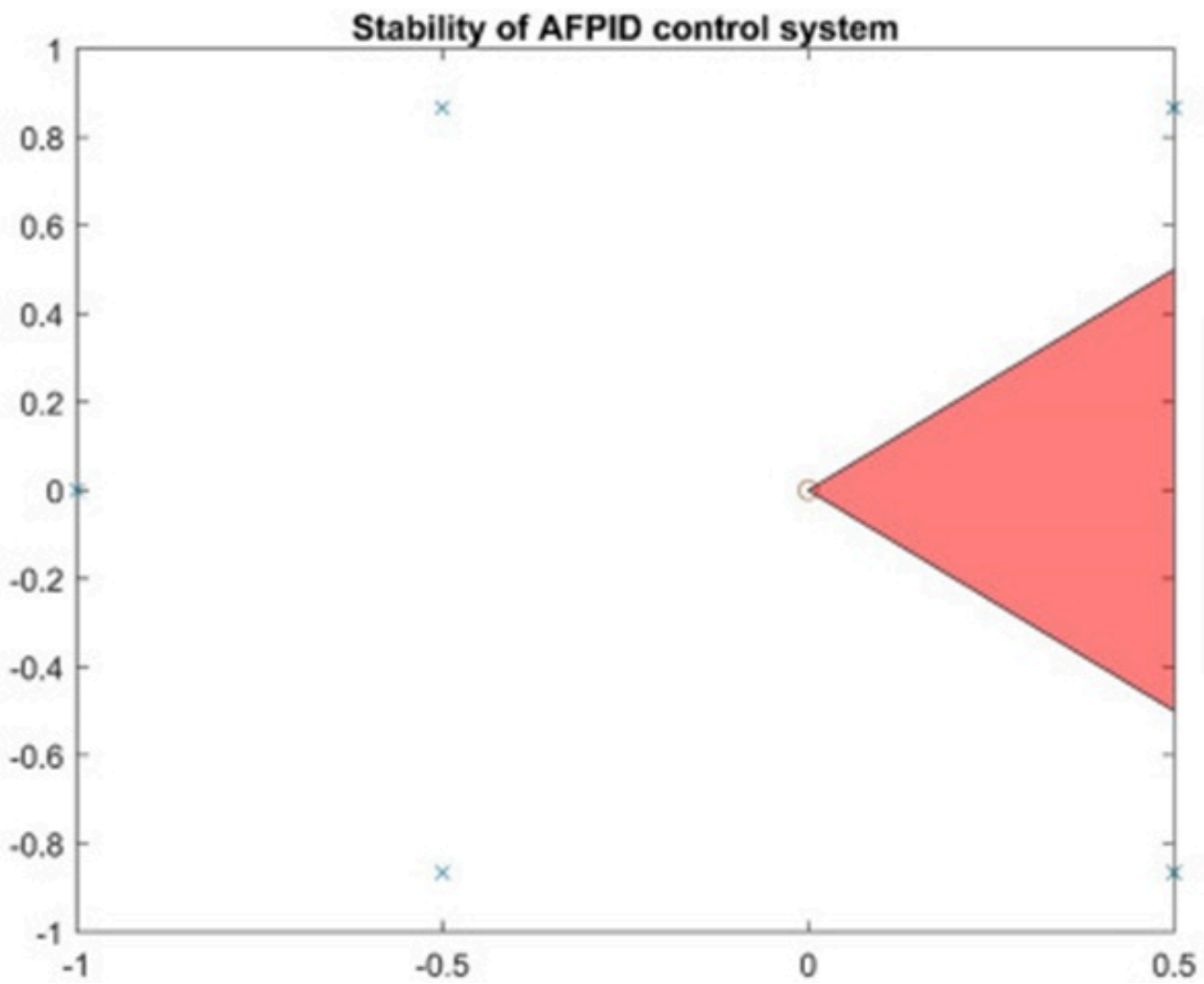


Figure 2. Stability region of the AFPID control system.

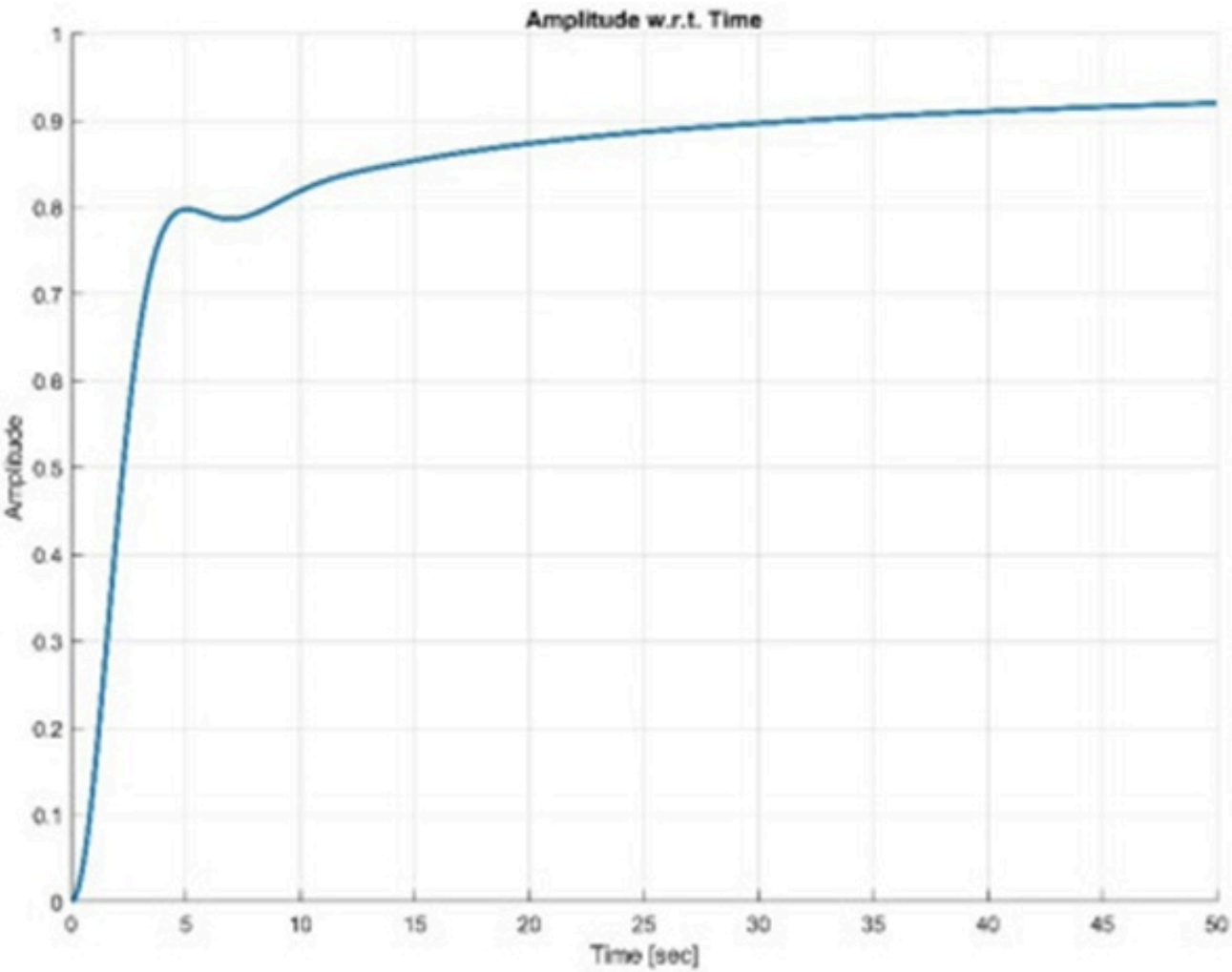


Figure 3. Amplitude variation with respect to time for the AFPID system.

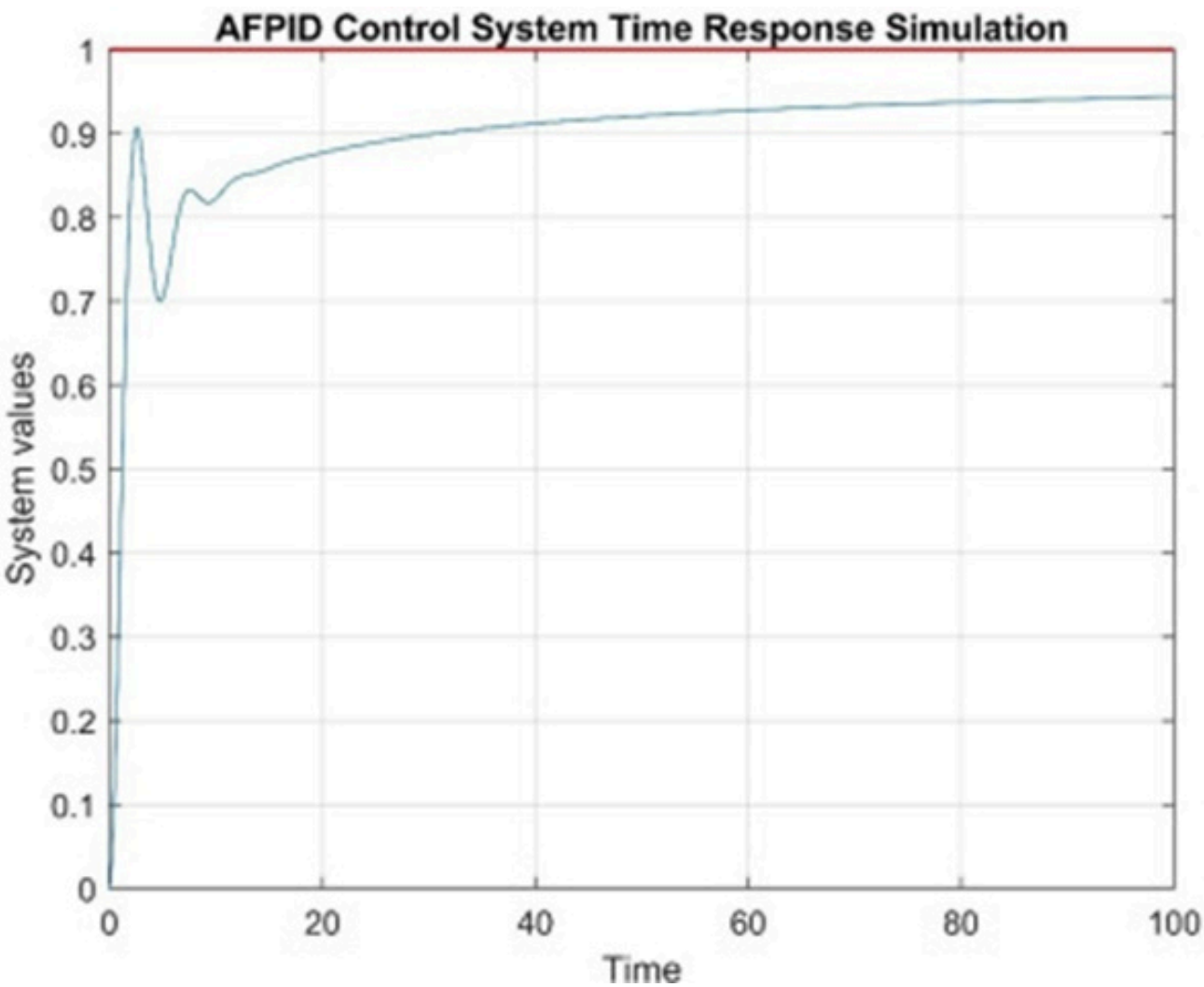


Figure 4. Time-domain responses of the AFPID system parameters.

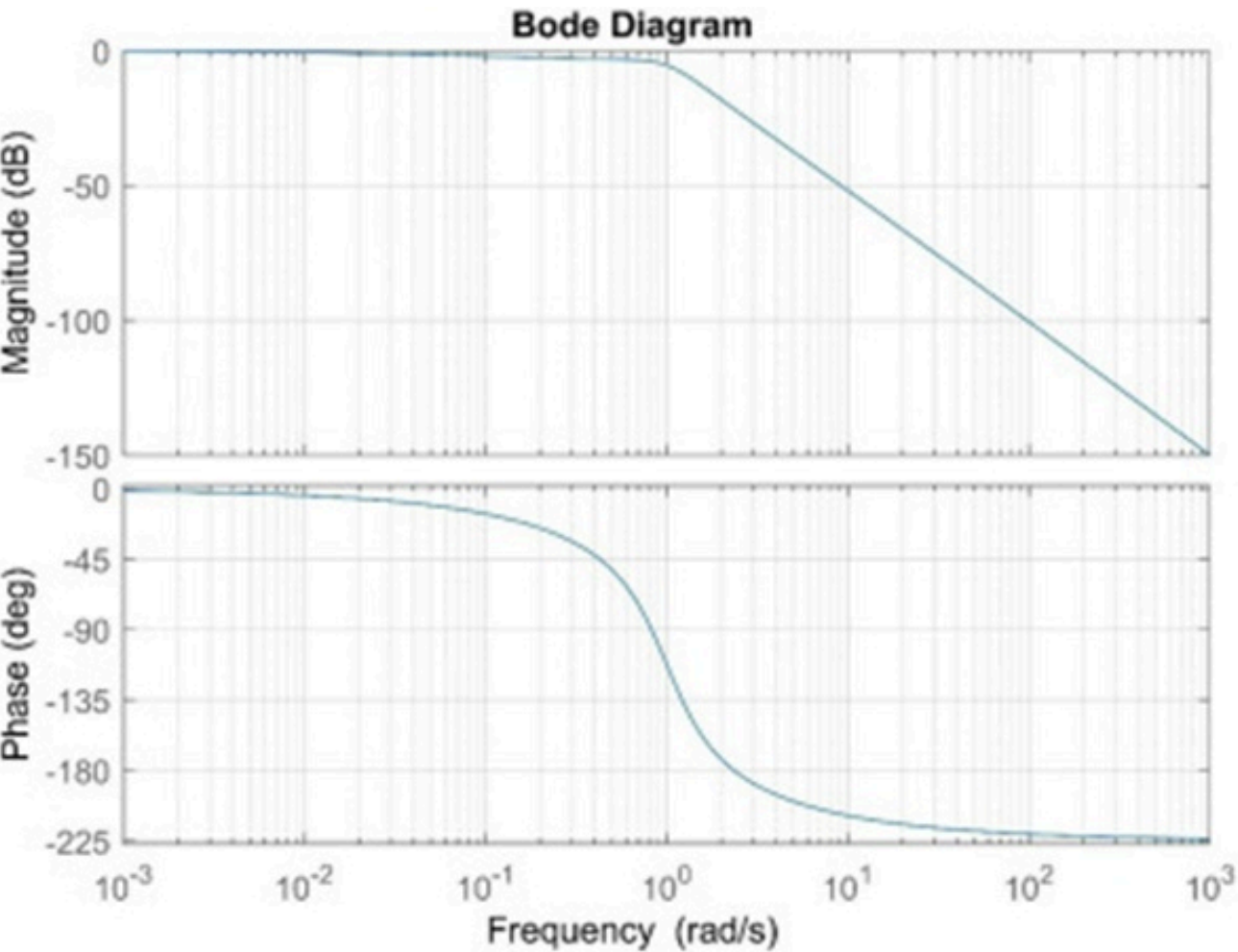


Figure 5. Bode diagram illustrating magnitude and phase variations versus frequency.

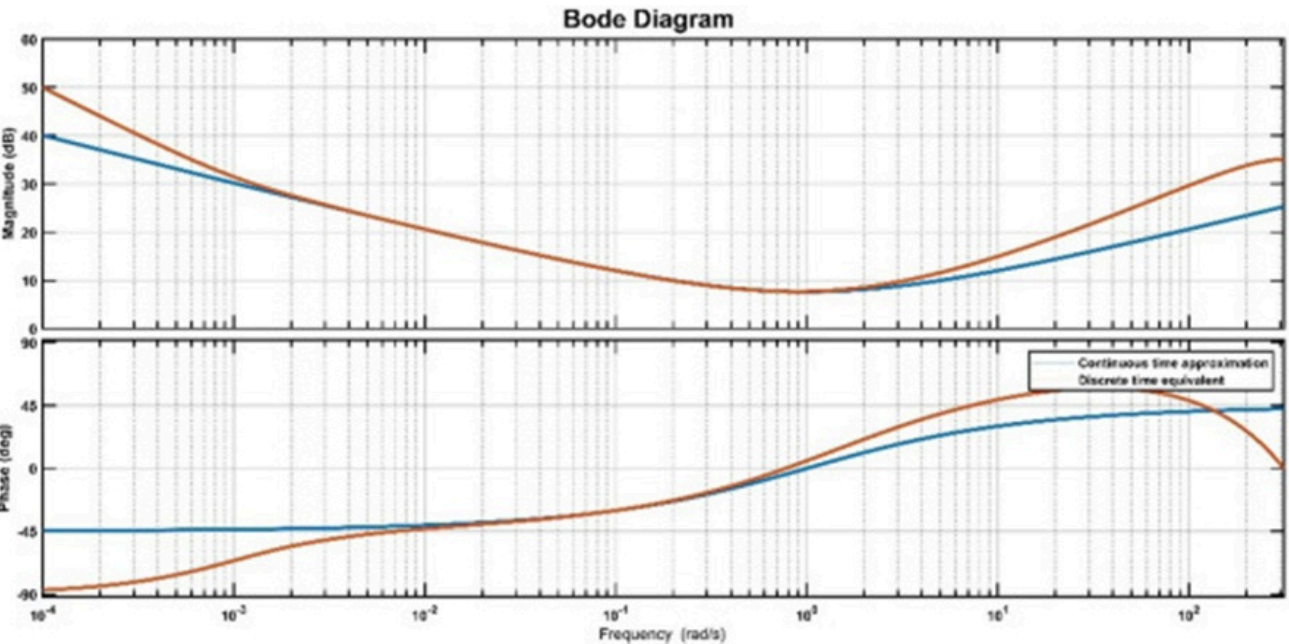


Figure 6. Bode diagram comparing magnitude and phase of continuous-time and discrete-time approximations.

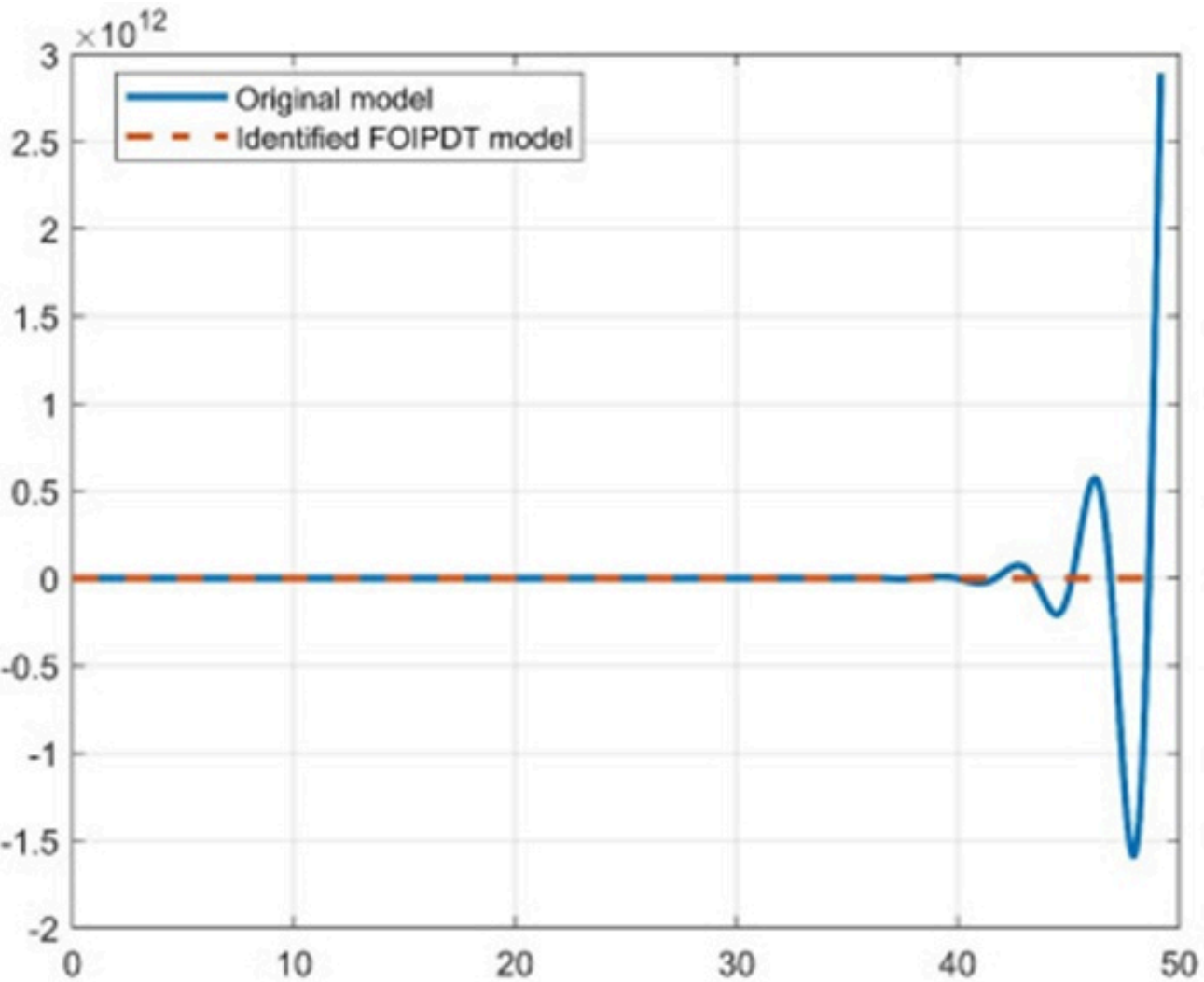


Figure 7. Performance comparison between the original model and AFPID models.

Table 1. Results obtained from two different PID models.

Simulation Aspect	ODE-Based Model [10]	AFPID Model
Simulation name	ODE-based model [10]	AFPID model
Simulation mode	Normal	Normal
Start time	0	0
Stop time	12	12
Solver name	Variable-Step	Variable-Step
Solver type	Variable Step Discrete	Variable Step Discrete
Initialization time (seconds)	2.64	0.87
Execution time (seconds)	0.48	0.15

Table 2. Comparison of gain responses across four controller types.

Parameter	Speed of Response	Stability	Accuracy
Conventional PID	Faster compared to systems without controllers	Generally stable	Better than systems without controllers
Hybrid Fuzzified PID (linear control surface)	Faster than traditional PID controllers	Steady-state error converges to zero	Superior to conventional PID controllers
Hybrid Fuzzified PID using Lookup Tables (nonlinear control surface)	Clearly faster than the other models examined in this study	Steady-state error approaches zero more rapidly	Slightly less accurate than the linear fuzzified PID variant
Fractional-order controllers	Response speed can be precisely adjusted using parameters α and β according to system dynamics	The stability region can be graphically simulated, with poles and zeros positioned along the segment $z \in (-1,1)$ in the z -plane	Highest accuracy achievable, depending on the selected step size and proper tuning of fractional orders

Table 3. Comparative overview of fractional and fuzzified PID controller studies.

Year	Reference	Description	Parameters / Approach	Results / Key Findings
October, 2010	Delavari et al. [16]	Adaptive fractional-order PID controller developed for robotic applications	Optimized using Genetic Algorithm	Numerical simulations demonstrated effective convergence of the controller
August, 2015	Baskonus and Bulut [10]	Numerical solutions investigated for selected fractional ordinary differential equations	Fractional Adams–Bashforth–Moulton method applied to two-dimensional surfaces	Both numerical and analytical solutions were simulated using appropriate parameter values
September, 2019	Bhardwaj and Bangia [14]	A fuzzified PID controller is proposed to enhance the efficiency of DC motors in electric battery vehicles	Fuzzy-integrated PID controller designed specifically for DC motors	Model formulation presented, along with analysis of stability and response speed
2024	Present study	Simulation-based fractional-order controller model for DC motor speed control	The fractional model was constructed and examined	Fractional-order model evaluated through stability analysis, phase response, and amplitude variations

Conclusion

The findings from the fractional-order PID controller case study exhibit consistency, thereby contributing to a deeper understanding of the interplay between chaotic phenomena and fractional calculus. Non-integer-order controllers applied to integer-order plants offer considerable flexibility, enabling adjustments to gain margins and phase characteristics that are not readily achievable with conventional integer-order (IO) control structures. The additional degrees of freedom introduced by fractional-order configurations—requiring fewer parameters to be tuned—have emerged as a powerful approach for designing high-performance control systems. Consequently, it has been demonstrated that fractional-order controllers can achieve robustness levels comparable to those of higher-order integer controllers by tuning only a few parameters.

Fractional-order PID controllers now provide a more accurate and industrially advantageous alternative compared to purely integer-order controllers. Comprehensive numerical methods for simulating fractional-order PID systems are presented. It has been shown that, through careful tuning, these controllers substantially improve the system's speed-control performance. From a theoretical standpoint, the presence of fractional orders in both differentiation and integration inherently results in infinite-dimensional linearized filters. Fractional-order PID (FOPID) controllers are therefore well-suited for a broad spectrum of real-world applications. Given the performance of integer-order PID control, it is reasonable to expect that adaptive fractional PID (AFPID) controllers will gain wider acceptance and deliver even greater efficiency.

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Conflict of interest

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None

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